

# **A Combined Time –Frequency Domain for Minimum Bit Error Rate Beamforming\***

**Dr.Waleed Abdallah Awad\*\***

**Dr.Yousef Abuzir\*\*\***

**Mr.Mohamad Khdair\*\*\*\***

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**\*\* Assistant Professor/ Al-Quds Open University/Palestine**

**\*\*\*Associate Professor/ Al-Quds Open University/Palestine**

**\*\*\*\*Lecturer/ Al-Quds Open University/Palestine**

## Abstract

Modern mobile communication systems are increasingly moving towards higher bit rates, thus requiring more efficient use of the available bandwidth. Multi-path fading, co-channel interference (CCI) and inter-symbol interference (ISI) are the main problems affecting mobile communications and limiting their capacity and performance. To overcome such problems, Orthogonal Frequency Division Multiplexing (OFDM), and Adaptive Antenna Array (AAA) are used to increase the overall performance.

In this paper, to overcome the problems mentioned before and to improve the communication system's performance we used two main methods Pre-FFT and Post-FFT with OFDM. In our case, to obtain the optimal weight sets two algorithms the least mean square (LMS) and Minimum Bit Error Rate (MBER) were used with each method. The results obtained after analysis and simulation show that the binary phase shift keying (BPSK) signaling based on MBER technique utilize the use of the antenna array elements efficiently compared to the (LMS) technique in both cases of pre-FFT and post-FFT. In addition, we proposed a combined scheme that uses analysis in the time and the frequency domains based on MBER algorithm. The combined scheme provided better performance compared to each individual scheme with a slight increase in complexity which makes it suitable for practical systems.

**Keywords:** MBER, OFDM, Pre-FFT, Post-FFT and LMS.

## ملخص

تتطور أنظمة الاتصالات الخلوية الحديثة بشكل متزايد فيما يتعلق بمعدل نقل البيانات، مما يتطلب زيادة كفاءة استخدام عرض النطاق الترددي المتوفر. تعتبر المشاكل التقنية المتعلقة بمشكلة استقبال الإرسال من عدة جهات مختلفة والتداخل بين إشارات القنوات المختلفة والتداخل بين الرموز من المشاكل الرئيسية التي تؤثر على الاتصالات الخلوية وتحد من قدرتها وإدائها. وللتغلب على هذه المشاكل، استخدمت تقنية التجميع التعامدي بتقسيم التردد (OFDM) وتقنية ضبط مصفوفة التكيف الهوائي لزيادة الأداء العام لأنظمة الهواتف الخلوية.

وللتغلب على المشاكل التي قمنا بذكرها سابقاً وتحسين أداء نظام الاتصالات استخدمنا طريقتين رئيسيتين وهما تقسيم الترددات المتعامدة pre-FFT و

post-FFT مع التجميع العامودي بتقسيم التردد. وتظهر النتائج التي تم الحصول عليها بعد التحليل والمحاكاة أن إشارة المفتاح التفاضلي لإزاحة الطور المبني على تقنية التجميع العامودي بتقسيم التردد يوظف استخدام عناصر مصفوفة التكيف الهوائي بفعالية مقارنة مع تقنية (LMS). وبالإضافة إلى ذلك، اقترحنا مخططاً مشتركاً يستخدم التحليل في الوقت ونطاقات التردد استناداً إلى خوارزمية (MBER). وقد وفر النظام الموحد أداءً أفضل بالمقارنة مع الخطط الفردية مع زيادة طفيفة في التعقيد مما يجعله مناسباً للنظم العملية.

## INTRODUCTION

Communication systems require high bandwidth due to the major advances in mobile devices that resulted in increasing the demand for high speed communication channels. However, high speed communications suffer from a number of main problems; one is multipath fading which results mainly due to reflected signals from objects in the communication channel between the transmitter and receiver, this reflection will make the reflected signals arriving at the receiver later than the original signal which causes delay spread. Moreover, in communication systems, another problem is inter-symbol interference (ISI). In general, it occurs within high speed communication channels in case the delay of the reflected signals is higher than the guard duration. Such problems can be solved by using Orthogonal Frequency Division Multiplexing (OFDM) which provides good performance in severe multipath fading channels. In addition, adaptive antennas array (AAA) can be used with OFDM to overcome co-channel interference (CCI) in wireless broadband communication systems. This can be helpful in certain mobile communication systems in which different cells use the same carrier frequency. The effect of CCI is reduced by OFDM with AAA which will subsequently result in more effective use of the available bandwidth.

However, Adaptive beamforming can separate transmitted signals on the same carrier frequency, because they are separated in the spatial domain. The beamforming can be realized by combining the signals received by the different elements of an antenna array and process them to form a single output. Therefore, by reducing ISI, Antenna Arrays can simplify the design of channel equalizer [1-5].

In OFDM systems, it is possible to apply AAA beamforming to time domain (Pre-FFT) or frequency domain (Post-FFT). Pre-FFT processing results in lower computations because

only one FFT operation is required; however, there is a slight performance degradation [1-2].

On the other hand, Post-FFT provides better performance at the cost of more complex computations that are required in this case because spatial processing of individual subcarriers is performed by applying the FFT operation on the received signal of each antenna [2].

## 1. Previous Work

- In [1] the LMS beamformer for Pre-FFT OFDM, they are presented without investigating several factors affecting the performance. The channel is assumed to be non-dispersive with additive Gaussian noise which is not a practical channel.
- In [2] the MBER beamformer for Pre-FFT OFDM are presented without investigating several factors affecting the performance. The channel is assumed to be non-dispersive with additive Gaussian noise which is not a practical channel.
- In [3], [4] the LMS beamforming algorithm for Pre-FFT OFDM system is applied on a frequency selective fading channel.
- An adaptive MBER beamforming was analyzed in [5] for single carrier modulation and in [2] for OFDM systems in additive Gaussian noise channel.
- In [6] both Post-FFT and Pre-FFT beamforming were considered. A maximum signal-to-noise ratio (max-SNR) Post-FFT beamformer in frequency domain and a switched-beam Pre-FFT beamformer in time domain were proposed.
- In [7] two beamforming algorithms were analyzed the low complexity Pre-FFT and a more efficient Post-FFT, analysis was based on determining the optimum weights that satisfy the LMS criterion and that satisfy the Recursive Least Squares (RLS) criterion.
- In [8], the LMS and MBER beamforming algorithms for Pre-FFT OFDM system were studied in a frequency selective fading channel, taking into account several factors affecting the performance of the two algorithms.
- In [9], it was shown that the diamond type pilot aided channel estimation has better performance when the channel is time-variant, in addition it was shown that the reduced number of pilot signals increases spectral efficiency.

- In [10], a novel Pre-FFT type OFDM adaptive array antenna called eigenvector combining is proposed. This system is a realization of a Post-FFT type OFDM adaptive array antenna through a Pre-FFT signal processing.
- In [11], this paper presented an optimum weight sets beamformer at time and frequency domain based on LMS algorithm. Different aspects of these systems were investigated such as different values of delay spread, angle of arrival of interfering sources and number of array elements.
- In [12], this paper presented the implementation of a joint Pre-FFT adaptive array antenna and Post-FFT Space Diversity Combining (AAA-SDC) scheme for mobile receiver. By applying a joint hardware and software approach, a flexible platform is realized in which several system configuration schemes can be supported.
- In [13], a smart antenna with Pre-FFT beamforming based on Eigen analysis and Post-FFT subcarrier diversity for broadband OFDM systems is proposed in order to achieve signal-to-noise ratio enhancement based on the use of two independent eigen beams in the Pre-FFT domain and maximum ratio combining in individual subcarriers in the Post-FFT domain.
- Finally, [14] presented a comparison of antenna array architectures for OFDM system using Pre-FFT and Post-FFT domain array processing, in addition, this paper investigated the dependency of the channel property for each array processing.

## 2. Main Contribution

The main contribution in this paper is to analyze the LMS and MBER algorithms of both Post-FFT and Pre-FFT in a practical channel model to find the best case in terms of performance and less error rate.

In this paper, we took into account the following factors affecting the performance of both Pre-FFT and Post-FFT beamformers: power of the noise and interferences, presence of a frequency selective channels in addition to directional interferences and angle spread.

It should be noted that MBER algorithm is more suitable for application in case of Post-FFT because it is less complex than LMS which makes it efficient in terms of system complexity.

A combined system which uses Pre-FFT and Post-FFT is proposed in this paper, this system was applied in case of LMS and MBER algorithms, it is shown that this combined scheme is superior compared to each of the individual Pre-FFT or Post-FFT schemes in both cases of LMS and MBER.

### 3. Organization

The rest of this paper is organized as follows: Section II describes the system model and beamforming schemes. Section III, describes adaptive algorithms. In Section IV simulation results are provided. Finally, conclusions are presented in section V.

## SYSTEM MODEL AND BEAMFORMING techniques

### A. OFDM System:

Figure 1 shows an OFDM system (Transmitter) using (AAA). Data bits at the transmitter are modulated using BPSK modulation. The resulting data is converted to a time-domain signal, then the cyclic prefix (CP) is added, the output of IFFT is transmitted to the channel.

The sample modulated by the  $k^{\text{th}}$  subcarrier of the  $m^{\text{th}}$  user is given by

$$x_m(k) = b_m(k) \quad 1 \leq m \leq M \quad 1 \leq k \leq K \quad (1)$$

where  $b_m(k) \in \{\pm 1\}$  for BPSK signals. The user

1 is assumed to be the desired user and the other sources are interfering users.

The following equations describes this process:

$$\bar{y}_m = \frac{1}{K} F^H \bar{x}_m \quad 1 \leq m \leq M \quad (2)$$

where

$$\bar{y}_m = [y_m(1) \ y_m(2) \ \dots \ y_m(K)]^T \quad (3)$$

$$\bar{x}_m = [x_m(1) \ x_m(2) \ \dots \ x_m(K)]^T \quad (4)$$

$$F = \begin{bmatrix} 1 & \frac{1}{K} & \dots & \frac{1}{K} \\ 1 & e^{-j\frac{2\pi(1)(1)}{K}} & \dots & e^{-j\frac{2\pi(1)(K-1)}{K}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi(K-1)(1)}{K}} & \dots & e^{-j\frac{2\pi(K-1)(K-1)}{K}} \end{bmatrix} \quad (5)$$

and  $H$  denotes the Hermitian transpose of a matrix. To add the CP,  $\bar{y}_m$  is cyclically extended to generate  $\tilde{y}_m$  by inserting the last  $v$  element of  $y_m$  at its beginning, i.e.

$$\tilde{y}_m = \begin{bmatrix} J_v \\ I_K \end{bmatrix} \bar{y}_m \quad (6)$$

where  $J_v$  contains the last  $v$  rows of a size  $K$  identity matrix  $I_k$ .

The channel used is a multipath channel model (frequency selective fading) and is assumed to include a maximum of  $L$  paths, also, it is assumed that the  $m^{\text{th}}$  source (desired or interference) and the receiving antenna array in the form of

$$h_m(k) = \sum_{l=0}^{L-1} \alpha_{m,l} \delta(k-l) \quad m = 1, \dots, M \quad (7)$$

where  $\alpha_{m,l}$  denotes a complex random number representing the  $l^{\text{th}}$  channel coefficient for the  $m^{\text{th}}$  source and  $\delta(\cdot)$  is delta function.

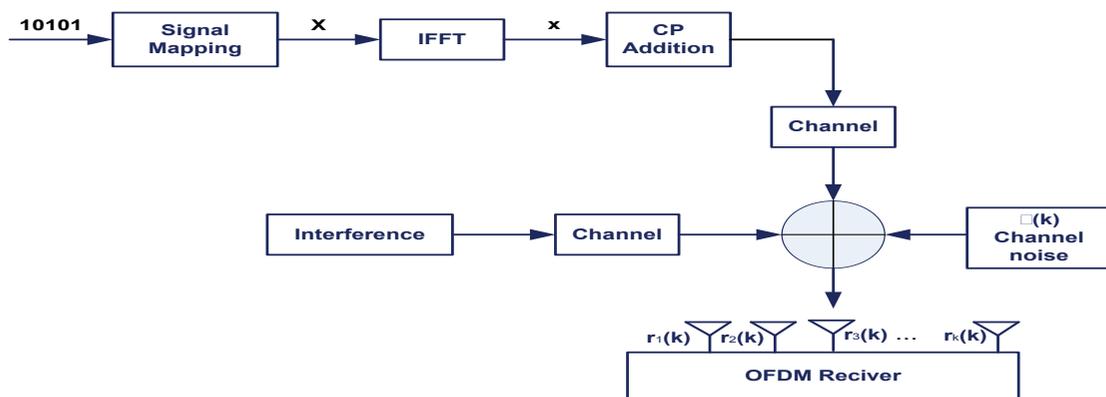


Fig. 1 The system model of OFDM transmitter

The signal at the receiver of an OFDM system, is described by equation (8), where CP is assumed to be longer than the channel length ( $v > L$ ), thus, received signal on the  $p^{\text{th}}$  antenna of a Uniform Linear Array (ULA) for one OFDM symbol can be writes as:

$$r_p(k) = \sum_{m=1}^M \sum_{l=0}^{L-1} \alpha_{m,l} \tilde{y}_m(k+v-l) e^{-j \frac{2\pi}{\lambda} (p-1)d \cos(\theta_{m,l})} + \eta_p(k) \quad 1 \leq p \leq P, \quad 1 \leq k \leq K \quad (8)$$

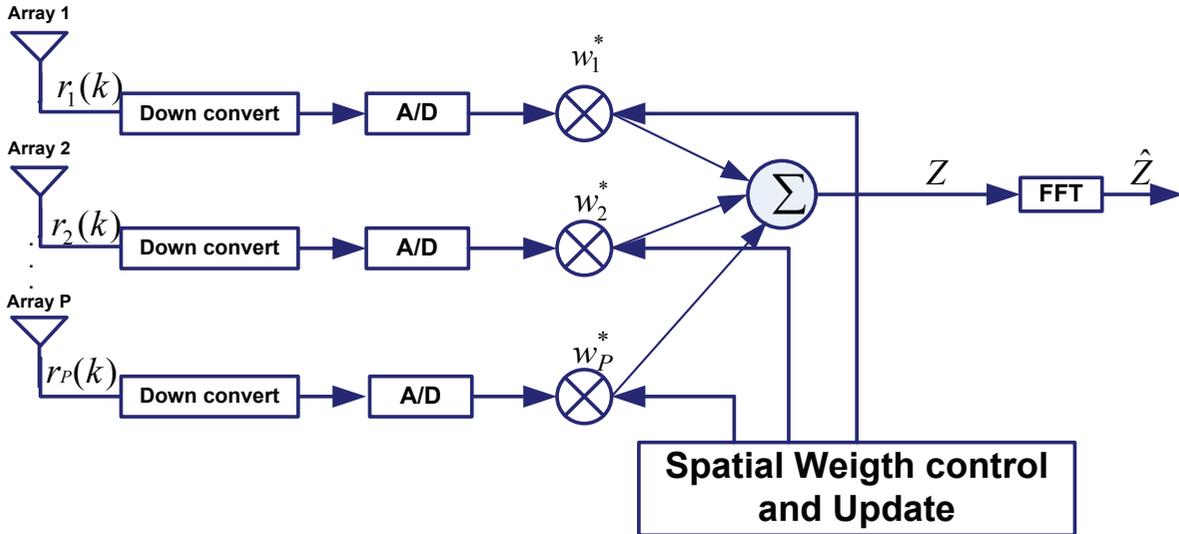
where  $\eta_n(k)$  represents channel noise entering the  $p^{\text{th}}$  antenna.  $\hat{e}_{m1}$  denotes the direction of

arrival (DOA) of the  $l^{\text{th}}$  path and  $m^{\text{th}}$  source.

**B. Pre-FFT Beamforming:**

In the receiver, the received signal with a spatial phase for  $p^{\text{th}}$  array element is multiplied by the  $p^{\text{th}}$  weight of adaptive beamformer ( $w_{pre}$ ), and the sum of this signals ( $\bar{Z}$ ) is transformed back into frequency-domain symbols ( $\hat{Z}$ ) by applying the FFT operator. This process can be written as:

$$\bar{Z} = W_{pre}^H \cdot \bar{r}(k) \quad (9)$$



**Fig.2.** Block diagram of time-domain (Pre-FFT) beamforming.

where

$$W_{pre} = [w_{pre1} \ w_{pre2} \ \dots \ w_{preP}]^T \quad (10)$$

$$\bar{r}(k) = [r_1(k) \ r_2(k) \ \dots \ r_k(k)]^T \quad (11)$$

$$\bar{Z} = [z(1) \ z(2) \ \dots \ z(K)]^T \quad (12)$$

$$\hat{Z} = F \cdot \bar{Z} \quad (13)$$

where  $\hat{Z}$  is the frequency-domain symbols (data and pilot), and is given by

$$\hat{Z} = [\hat{z}(1), \hat{z}(2), \dots, \hat{z}(K)]^T \quad (14)$$

and  $\hat{z}(k)$  denotes the corresponding received sample at the  $k^{\text{th}}$  subcarrier.

Known pilot symbols are sent to implement LMS and then are compared with their known values to generate an error signal that is used to correct data bit errors received on the same channel. If there are a total of  $Q$  pilot symbols in every OFDM symbol then we define two  $K \times 1$  vector  $d_q$  and  $Z_q$  such that, the  $k$ th element of  $d_q$  is zero if  $k$  is a data subcarrier and is the known pilot value if  $k$  is a pilot subcarrier. Similarly, the  $k$ th element of  $Z_q$  is zero if  $k$  is a data subcarrier

and is the received pilot value if  $Q$  is a pilot subcarrier [9]. Therefore, the error signal in frequency domain is given by:

$$E_q = d_q - Z_q \quad (15)$$

The error signal, in turn, has to be converted to time domain for the Pre-FFT weight adjustment algorithm. Therefore,

$$\bar{e} = \frac{1}{K} F^H E_q \quad (16)$$

where  $\bar{e}$  is the vector of error samples in time domain.

$$\bar{e} = [e(1)e(2) \dots e(K)]^T \quad (17)$$

Consequently, the Pre-FFT weights are updated as shown later in Adaptation Algorithms section in this paper.

### C. Post-FFT Beamforming:

As shown in Fig. 2 (block diagram of the Post-FFT beamforming), the received time domain signal of each antenna is first converted to frequency domain, then beamforming is performed on each subcarrier [1-2],[9]. If  $R_{k,p}$  denotes the  $k^{th}$  subcarrier of the  $p^{th}$  antenna, then the (frequency-domain) output signal of  $k^{th}$  subcarrier is given by:

$$Y(k) = \sum_{p=1}^P w_{k,p}^* R_{k,p} \quad 1 \leq k \leq K \quad (18)$$

In equation (18),  $w_{k,p}$  represents the weight associated with  $R_{k,p}$ . In Fig. 2 one weight is applied to every subcarrier, because we assume that all subcarriers are pilot. Since there exist only a few pilots in each OFDM block, every group of adjacent data subcarriers are clustered under one pilot symbol and the weight of that pilot symbol is applied to all data subcarriers in the cluster [1-2],[9].

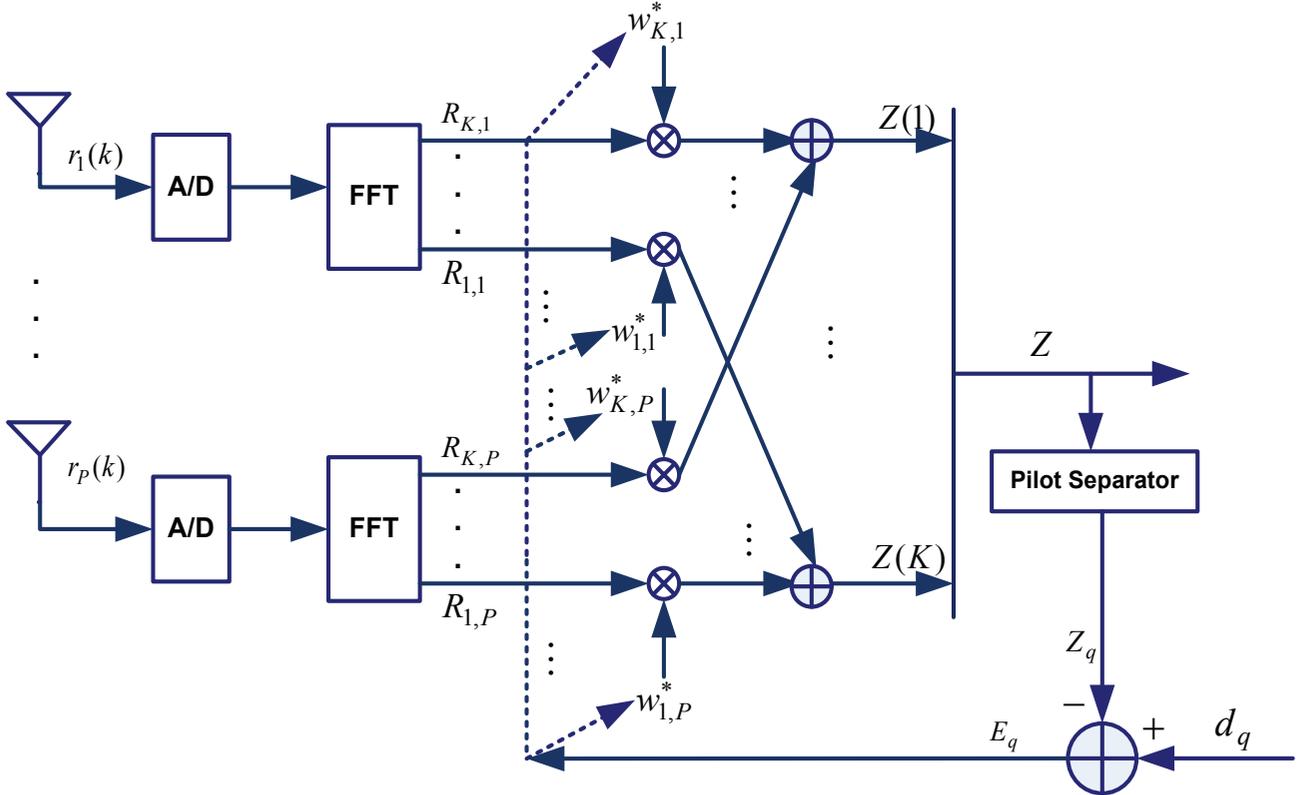


Fig. 2:

Block diagram of frequency-domain (post-FFT) beamforming

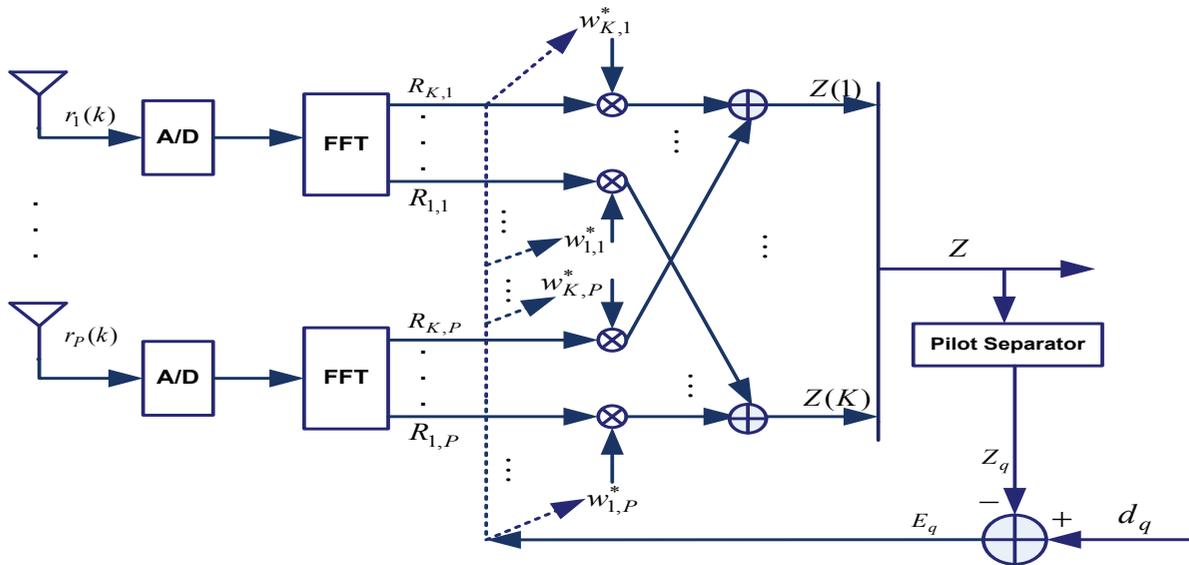
**D. Combined Pre-FFT / Post-FFT Scheme**

The purpose of this method is to utilize the benefits of each scheme in order to achieve superior performance. However, the increased complexity of the resulting system is a drawback. In earlier work, different combined schemes were proposed [10],[13], in which the weights were adjusted using complex algorithms, thus requiring more computations and in turn degrading the benefit of the overall system.

Our proposed combined system is shown in figure (3). The system works by using weights for the received signal in both time and frequency domains. It is possible to use the system as Post-FFT only by setting Pre-FFT weights to one; similarly, the system can be used as Pre-FFT only

by setting Post-FFT weights to one. The received time-domain signals are first multiplied by Pre-FFT weights  $W_{pre,k}$  and are then passed through FFT blocks. The frequency domain signals  $R_{p,k}$  are subsequently multiplied by Post-FFT weights  $w_{p,k}$ .

Since only the error signal of the pilots in frequency domain is available, this signal is used for updating both the Pre-FFT and Post-FFT weights simultaneously. Although the same error information is used in both schemes, they produce different results because of the different methods used in each scheme to compensate for the channel. Post-FFT weights in the combined scheme are update based on equation (20). Whereas Pre-FFT weights can also be obtained in frequency domain by using the output resulting from pilot signals:  $Y(p)$ .



**Fig. 3**

The proposed combined Pre-FFT / Post-FFT beamforming

**ADAPTIVE ALGORITHMS**

The adaptive beamforming algorithms are used to update the weight vectors periodically to track the signal source in a time varying environment. This is done by adaptively modifying the system’s antenna pattern so that nulls are generated in the directions of the interference sources.

**A. Least Mean Square (LMS) Algorithm:**

The LMS algorithm can be defined as a

method that randomly implements the steepest descent algorithm. Successive corrections to the weight vector in the direction of the negative of the gradient vector eventually lead to the LMS, at which point the weight vector assumes its optimum value.

Consequently, the Pre-FFT weights are updated using the following LMS algorithm [4]

$$W(k) = W(k - 1) + 2\mu \cdot r(k) \cdot e^*(k) \quad 1 \leq k \leq K \quad (19)$$

$\mu$  represents the step size parameter, whereas \* refers to complex conjugate. The last update at the end of each OFDM block ( $W(K)$ ) is used as the initial value of the next block. As the step size increases, the mean square error also increases.

Then the Post-FFT weights are updated with the following LMS equation for  $(q+1)$ -th OFDM block.

$$W_k(q+1) = W_k(q) + 2\mu \cdot R_k \cdot E_k^*(q) \quad (20)$$

The frequency –domain output signal of  $k$ -th subcarrier is given by

$$Y(k) = \sum_{p=1}^P w_{k,p} R_{k,p} \quad 1 \leq k \leq K \quad (21)$$

where  $E_q$  denotes the error signal of  $q$ -th pilot subcarrier.

Note that the frequency domain error signal  $E_k(q)$  is employed in equation (15),

Post-FFT weights in the combined scheme of figure (3) are update based on equation (20). For updating the Pre-FFT weight we can also use frequency domain signal.

which implies the following LMS equation

$$W_{pre}(k+1) = W_{pre}(k) + 2\mu \cdot Q(k) \cdot E_k^* \quad 1 \leq k \leq K \quad (22)$$

where

$$Q_{k,p} = w_{k,p}^* \cdot D_{k,p} \quad 1 \leq k \leq K \quad (23)$$

$$D_{k,p} = \frac{R_{k,p}}{w_{pre_p}^*} \quad 1 \leq k \leq K \quad (24)$$

### B. Minimum Bit Error Rate (MBER) Algorithm

MBER algorithm is used to obtain the optimum weight set. This algorithm can be implemented with Pre-FFT and Post-FFT. The block diagram of the Pre-FFT beamforming is shown in Fig. (2), the estimate of the transmitted bit  $b_1(k)$  is given by

$$\hat{b}_1(k) = \begin{cases} +1, & \text{Re}(\hat{z}(k)) > 0 \\ -1, & \text{Re}(\hat{z}(k)) \leq 0 \end{cases} \quad (25)$$

where  $\text{Re}(\hat{z}(k))$  denotes the real part of  $\hat{z}(k)$ .

The theoretical MBER solution for the Pre-FFT OFDM beamformer is obtained in [5-8] where, the channel is assumed to be non-dispersive with additive Gaussian noise. The error probability (BER cost function) of the frequency domain signal of the beamformer is given by:

$$P_E(W) = \text{Prob}\{\text{sgn}(b_1(k))\text{Real}(\hat{z}(k)) < 0\} \quad (26)$$

where  $\text{sgn}(\cdot)$  is the sign function. The weight vector that minimize the BER is then defined as:

$$W = \arg \min_W P_E(W) \quad (27)$$

From equation (25), define the signed decision variable

$$\begin{aligned} \hat{z}_s(k) &= \text{sgn}(b_1(k))\text{Re}(\hat{z}(k)) \\ &= \text{sgn}(b_1(k))\text{Re}(\hat{z}'(k)) + \eta'(k) \end{aligned} \quad (28)$$

where

$$\hat{z}'(k) = W^H [\bar{r}(k) - \eta(k)] F(k) \quad (29)$$

and

$$\eta'(k) = \text{sgn}(b_1(k))\text{Re}(W^H) \eta(k) F(k) \quad (30)$$

$\hat{z}_s(k)$  is the error indicator for the binary decision, when it is positive, then the decision is correct, else an error occurred,  $F(k)$  is the  $\eta'(k)$   $k^{\text{th}}$  column of F. When F is unitary matrix,  $\eta'(k)$  remains Gaussian with zero mean and variance

$$\sigma_n^2 \cdot W^H W$$

The conditional probability density function (pdf) given the channel coefficients  $\alpha_{m,l}$  of the error indicator,  $\hat{z}_s(k)$ , is a mixed sum of Gaussian distributions [6], i.e.,

$$p_z(\hat{z}_s) = \frac{1}{K\sqrt{2\pi}\sigma_n\sqrt{W^H W}} \cdot \sum_{k=1}^K \exp\left(-\frac{(\hat{z}_s - \text{sgn}(b_1(k))\text{Re}(\hat{z}'_s(k)))^2}{2\sigma_n^2 W^H \cdot W}\right) \quad (31)$$

This is a good indicator for the beamformer's BER performance, because deriving a closed form for the average error probability is not an easy process. To update the weight vector, the gradient conditional error probability is used, provided that the channel coefficients  $\alpha_{m,l}$  of the beamformer

$P_E(W)$ , is given by [5-8].

$$P_E(W) = \frac{1}{K\sqrt{2\pi}\sigma_n\sqrt{W^H W}} \cdot \sum_{k=1}^K \int_{q_k(W)}^{\infty} \exp\left(-\frac{u^2}{2}\right) du = \frac{1}{K} \sum_{k=1}^K Q(q_k(W)) \quad (32)$$

where

$$u = \frac{(\hat{z}_s - \text{sgn}(b_1(k))\text{Re}(\hat{z}'_s(k)))}{\sigma_n\sqrt{W^H W}} \quad (33)$$

where  $Q(\cdot)$  is the Gaussian error function.

$$q_k(W) = \frac{\text{sgn}(b_1(k))\text{Re}(\hat{z}'_s(k))}{\sigma_n\sqrt{W^H W}} \quad (34)$$

In OFDM system, we assumed that there are pilot signals in every symbol to carry out channel estimation. [1-4].

The pilot signals are also used to adaptively update the weight vector of the beamformer. We can write both the transmitted signal vector of desired user  $\bar{x}_{1p}$  and the received pilot signal vector  $\hat{z}_p$  in the frequency domain as follows

$$\bar{x}_{1p} = [x_1(1), 0, \dots, x_1(\Delta p + 1), 0, \dots, x_1((K_p - 1)\Delta p + 1), 0, \dots] \quad (35)$$

$$\hat{z}_p = [\hat{z}(1), 0, \dots, \hat{z}(\Delta p + 1), 0, \dots, \hat{z}((K_p - 1)\Delta p + 1), 0, \dots] = W^H \bar{R} F_p$$

where

$$F_p = \begin{bmatrix} 1 & 0 & \dots & 1 & \dots & 1 & 0 \dots \\ 1 & 0 & \dots & e^{-j2\pi(1)(\Delta p)/K} & \dots & e^{-j2\pi(1)(K_p-1)\Delta p/K} & 0 \dots \\ \vdots & 0 & & \vdots & \ddots & \vdots & 0 \dots \\ 1 & 0 & \dots & e^{-j2\pi(K-1)(\Delta p)/K} & \dots & e^{-j2\pi(K-1)(K_p-1)\Delta p/K} & 0 \dots \end{bmatrix} \quad (36)$$

$\bar{R} = [\bar{r}(1) \bar{r}(2) \dots \bar{r}(K)]$ ,  $\Delta p$  and the frequency

spacing between consecutive pilot symbol and the number of pilot symbols that are inserted in OFDM are respectively represented by  $K_p$

It is also assumed that the first pilot symbol is inserted at the first sub-channel.

The method of approximating a conditional pdf known as a kernel density or Parzen window-based estimate [6-8], it is used to estimate the conditional error probability given the channel coefficients  $\alpha_{m,l}$  that is used on OFDM systems.

Also, given a symbol of  $K_p$  training samples  $\{\bar{r}(k) b_1(k)\}$ , a kernel density estimate of the conditional pdf based on the channel coefficients  $\alpha_{m,l}$  at pilot locations is given by

$$\hat{p}(\hat{z}) = \frac{1}{2\pi K_p \rho_\eta^2 W^H W} \sum_{K_p=1}^{K_p} \exp\left(-\frac{|\hat{z} - \hat{z}(k)|^2}{2\rho_\eta^2 W^H W}\right) \quad (37)$$

where the kernel width  $\rho_\eta$  is related to the noise standard deviation  $\sigma_\eta$ . From this estimated pdf, the estimated BER is given by:

$$\hat{P}_E(W) = \frac{1}{K_p} \sum_{k=0}^{K_p-1} Q(\hat{q}_k(W)) \quad (38)$$

where

$$\hat{q}_k(W) = \frac{\text{sgn}(b_1(k \times \Delta p + 1))\text{Re}(W^H \bar{R} F_p(k \times \Delta p + 1))}{\rho_n \sqrt{W^H W}} \quad (39)$$

And  $F_p(k \times \Delta p + 1)$  is the  $(k \times \Delta p + 1)^{th}$  column of  $F_p$ , from this estimated conditional pdf given the channel coefficients  $\alpha_{m,l}$ , the gradient of the estimated BER is given by [5-8]

$$\nabla \hat{P}_E(W) = -\frac{1}{\sqrt{2\pi\rho_n}\sqrt{W^H W}} \sum_{k=0}^{K_p-1} \exp\left(-\frac{(\text{Re}(\hat{z}(k \times \Delta p + 1)))^2}{2\rho_n^2 W^H W}\right) \times \text{sgn}(b_1(k \times \Delta p + 1)) \bar{R} F_p(k \times \Delta p + 1) \quad (40)$$

Now a block-data adaptive MBER algorithm Table (1) is obtained by the gradient of  $\hat{P}_E(W)$ .

The vector representing the optimum weight for each W OFDM symbol can be found using the steepest- descent gradient algorithm [6].

$$\nabla \hat{P}_E(W) = -\frac{1}{\sqrt{2\pi\rho_n}} \exp\left(-\frac{(\text{Re}(\hat{z}(k \times \Delta p + 1)))^2}{2\rho_n^2}\right) \times \text{sgn}(b_1(k \times \Delta p + 1)) \bar{R} F_p(k \times \Delta p + 1) \quad 1 \leq k \leq K_p \quad (41)$$

**Table 1.**

**MBER algorithm summary.**

Initialization
$i = 1, \mu = .008, \text{Block size } K = 64$
- Calculate variance of noise $\sigma_n$
- Initial weight vector $W = .01 * \text{ones}(N,1)$
Outer loop (1: floor (all bits/Block))
Form a block of data from the received signals.
Inner loop (while $k < K_p$ )
- Calculate the gradient matrix from equations (41).
- Update the weight matrix as $W(k) = W(k-1) + \mu \nabla \hat{P}_E$
from equation (42), (43), (46).
- Make the solution normal $W(k+1) = W(k+1) / \ W(k+1)\ $
- end of inner loop
- Determine the detected signals.
- Increment the block number $i = i + 1$
- end of outer loop

Table 1 shows a summary of the proposed MBER algorithm. The main algorithm parameters are set at the beginning. Then we defined the two loops in the algorithm: The outer loop is for each block of data, and the inner loop is repeated over the same block of data until a certain number of iterations is reached (e.g., 16 iterations), the norm of the gradient vector is sufficiently small. We computed the BER cost function and the gradient matrix (41). Then, we computed the weight update vector from equations (42), (43) and (46). At the end of the inner loop, we determined the detected signal by multiplying the computed weight vector by the received signal to use it to calculate the BER cost function again and increment the inner loop iteration index. The inner loop iterates until any of the stop criteria is reached. After that, we go back to the main loop and form another block of data, and so on. These processes iterate until we finish all the incoming data.

This algorithm indicates that W weight vector can be updated  $K_p$  times in one OFDM symbol. So, we lessen the complexity and consequently the update equation is given by

$$W(k+1) = W(k) - \mu \nabla \hat{P}_E(W) \\ = W(k) + \frac{\mu}{\sqrt{2\pi\rho_n}} \cdot \exp\left(-\frac{(\text{Re}(\hat{z}(k \times \Delta p + 1)))^2}{2\rho_n^2}\right) \times \text{sgn}(b_1(k \times \Delta p + 1)) \bar{R} F_p(k \times \Delta p + 1) \quad (42)$$

where  $\mu$  is a step size.

Then the Post-FFT weights are updated with the following MBER equation for  $(q+1)$ -th OFDM block.

$$W_k(q+1) = W_k(q) + 2\mu \cdot R_k \cdot \Delta \hat{P}_E(W) \quad (43)$$

where

$$R_k = [R_{m,1} \ R_{m,2} \dots \ R_{m,p}]^T \quad (44)$$

The frequency –domain output signal of  $k$ -th subcarrier is given by

$$Y(k) = \sum_{p=1}^P w_{k,p} R_{k,p} \quad 1 \leq k \leq K \quad (45)$$

Post-FFT weights in the combined scheme of figure (4) are update based on equation (20). For updating the pre-FFT weight we can also use frequency domain signal.

which implies the following MBER equation

$$W_{pre}(k+1) = W_{pre}(k) + 2\mu \cdot Q(k) \cdot \Delta \hat{P}_E(k) \quad 1 \leq k \leq K \quad (46)$$

where

$$Q_{k,p} = w_{k,p}^* \cdot D_{k,p} \quad 1 \leq k \leq K \quad (47)$$

$$Q(k) = [Q_{k,1} \quad Q_{k,2} \dots \quad Q_{k,P}]^T \quad (48)$$

then,

$$Y(k) = W_{pre}^H Q(k) \quad (49)$$

$$D_{k,p} = \frac{R_{k,p}}{w_{pre,p}^*} \quad 1 \leq k \leq K \quad (50)$$

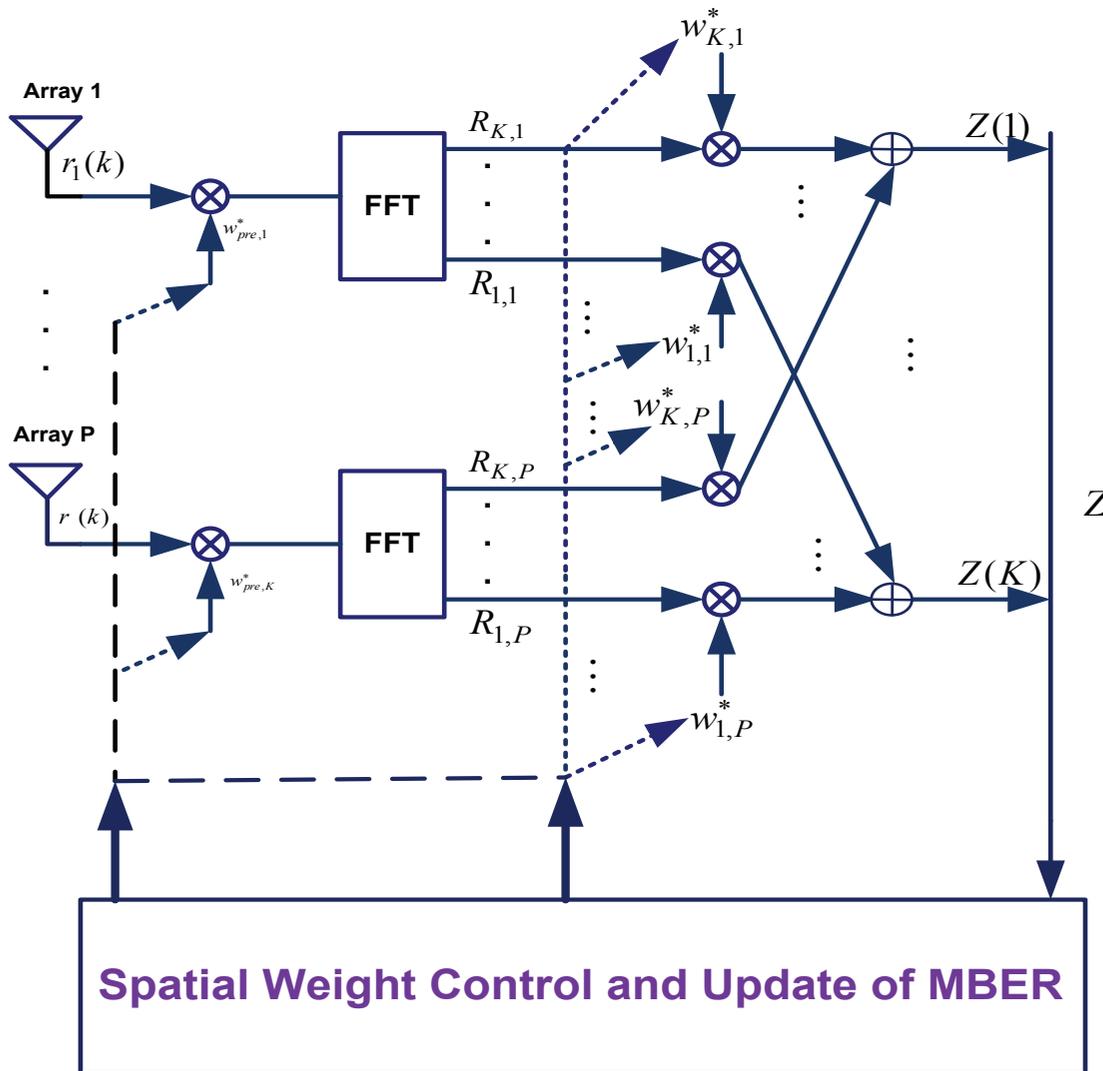


Fig. 4.

The proposed combined Pre-FFT / Post-FFT beamforming (MBER)

It can be shown in figure (4) that the computational complexity is not much increased compared to Post-FFT.  $W_{pre}(w)$  is applied to all time samples of the next OFDM block, which means that the computational load of the combined

scheme is equal to the Post-FFT plus the Pre-FFT. However, the computational load for Pre-FFT is much less than the Post-FFT, so the overall load is not much increased but the performance is enhanced significantly.

## SIMULATION RESULTS

In this section, simulations are conducted to evaluate the performance of the proposed adaptive beamforming for the LMS and MBER algorithms in a variety of channel conditions. We assumed an OFDM system perfectly synchronized, with a CP length larger than the channel length with 64 subcarriers (pilot + data), BPSK modulation scheme, one desired source and two interferences with equal powers. Some sources, such as the desired and interference sources were placed at a fixed angle,  $70^\circ$ ,  $20^\circ$ , and  $120^\circ$ , respectively. Normalized channels with different length and real coefficients were assumed of 0.864, 0.435, 0.253 and 0 for all sources, and an angle spread of  $\pm 15^\circ$  [1, 8]. We also assumed that pilots have to be distributed in a unified form in the OFDM block. We also considered the first subcarrier in every cluster as a pilot.

The following table shows the system specifications

SNR	[0-14] dB
Step size ( $\mu$ )	0.008
Kernel radius ( $\rho$ )	$=1 \sigma_n$
Number of subcarriers	64
OFDM symbol time	1000 symbol periods
Guard time	16 symbol periods
Angle spread	$\pm 15^\circ$
Number of users	3
BS receive antennas	6
Antenna spacing	$d = \lambda/2$
Channel coefficients	[0.864 0.435 0.253]
Modulation	BPSK
Comb -pilot	16
Channel	synchronous Block Rayleigh fading
Noise	Complex AWGN
Computational Complexity of LMS	$O(M)$
Computational Complexity of MBER	$O(M)$

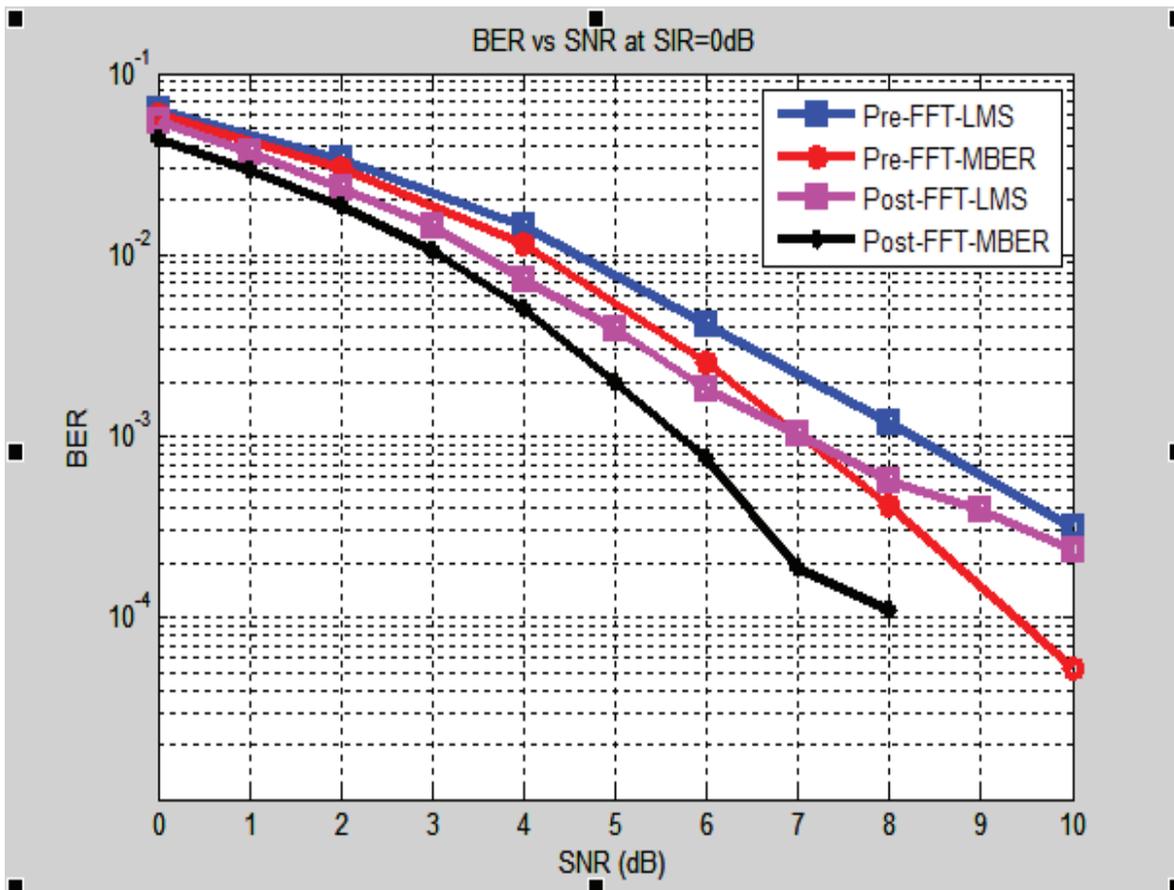


Fig.5.

Bit error rate performance (6 antenna elements, 3 users, SIR= 0 dB)

Fig. 5 shows the BER plots of Pre-FFT and Post-FFT method as a function of input SNR and SIR=0 dB. It is shown that the performance of the Post-FFT scheme is better than the Pre-

FFT method. The BER of the Post-FFT MBER beamformer performs better than LMS under moderate SNR.

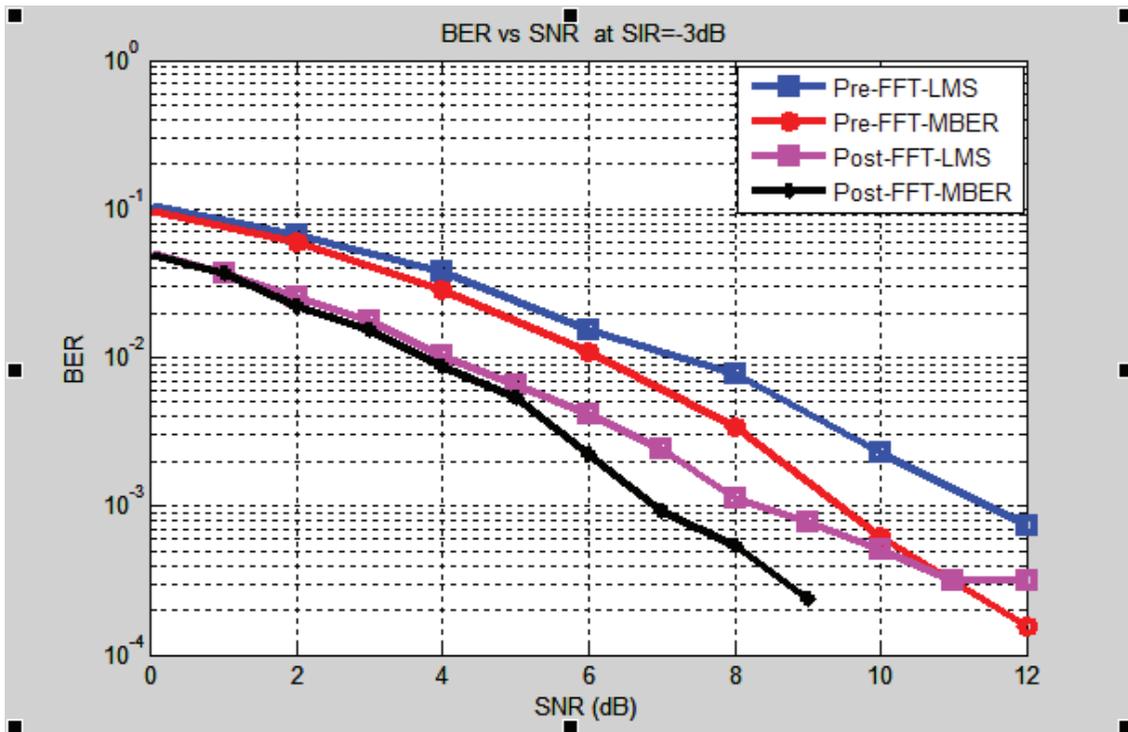


Fig.6.

Bit error rate performance (6 antenna elements, 3 users, SIR= -3dB)

Fig.6. shows a similar comparison to that shown in Fig.5 but for SIR= -3 dB. Similarly, in this case, the performance of the Post-FFT is

better than the Pre-FFT. Also, it is observed that the BER of the Post-FFT MBER beamformer is superior to that of LMS under moderate SNR.

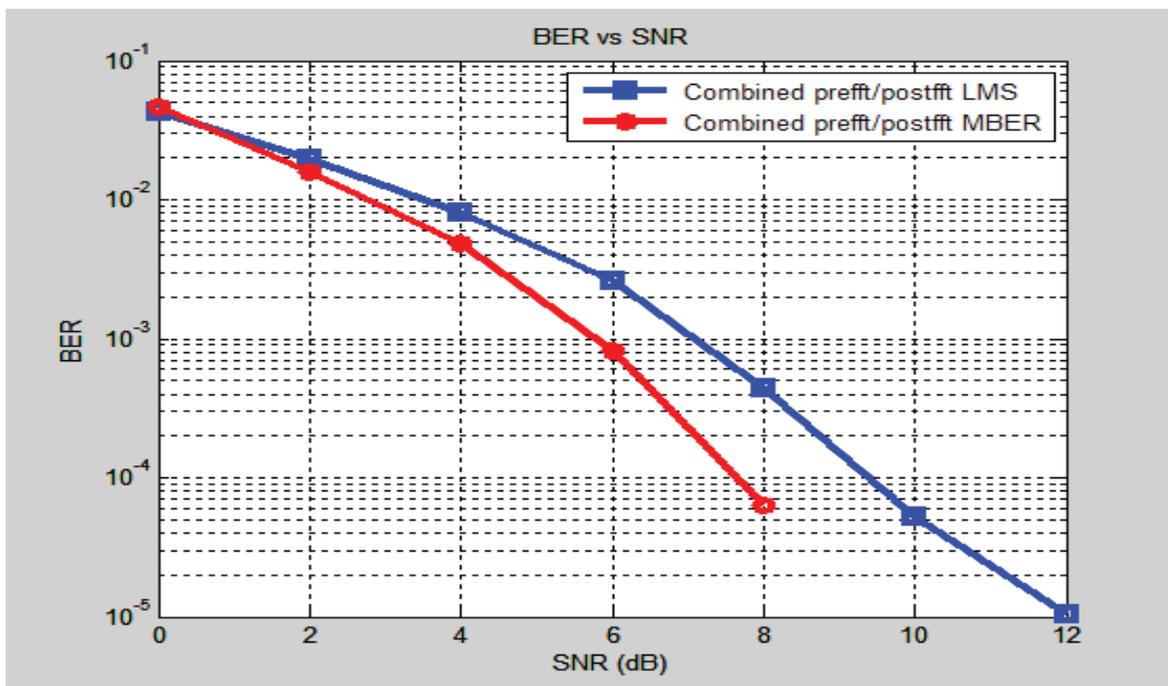


Fig.7.

Bit error rate performance (6 antenna elements, 3 users, SIR= -3dB).

Fig.7 shows a comparison between the combined LMS scheme and the proposed combined MBER scheme. The performance of the combined Post-FFT / Pre-FFT MBER is better

than the combined Post-FFT /Pre-FFT LMS. Also, it is observed that the BER of the combined Post-FFT/Pre-FFT MBER beamformer is superior to that of LMS under moderate SNR.

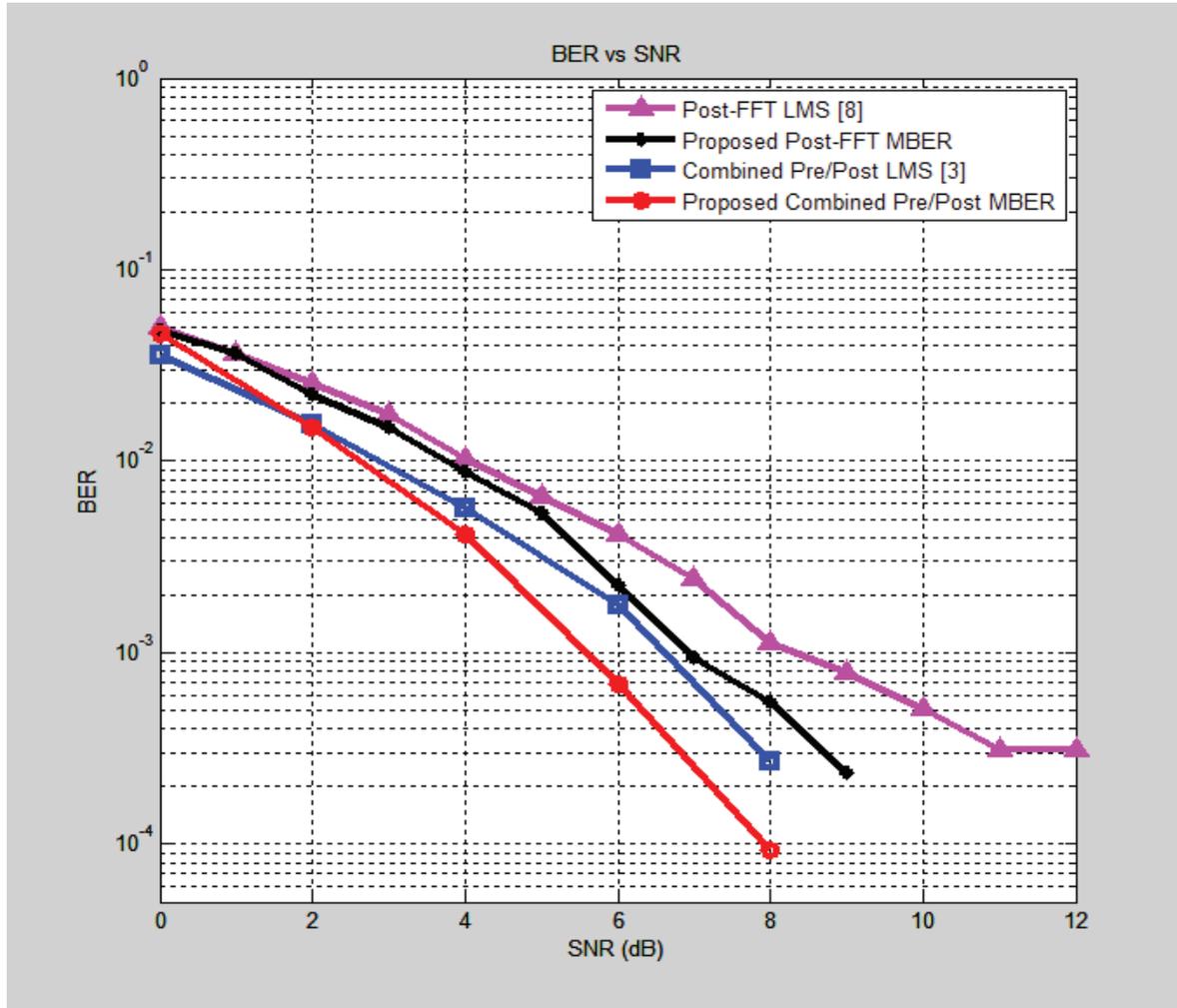


Fig 8.

Bit error rate performance (6 antenna elements, 3 users, SIR= -3dB).

Fig.8 shows a comparison between the proposed Post-FFT for MBER and Post-FFT for LMS in [8]. At the same time, we compare the results with the combined Pre-FFT/Post-FFT for LMS in [3] and the proposed combined Pre-FFT/Post-FFT for MBER. Results show that the combined schemes provide better performance compared to individual schemes, also, the MBER combined scheme has provided the best results in case of BER. It should be noted that while the combined LMS scheme has outperformed

individual Post-FFT schemes but this was done at the cost of significant increase in system complexity, however, it is the case of combined MBER the performance that was superior with a slight increase in complexity.

Fig. 9 and Fig. 10 illustrate the convergence performance of MBER Pre-FFT and Post-FFT beamformer for different values of SNR. Compared to Pre-FFT, the Post-FFT results indicated better performance and faster convergence.

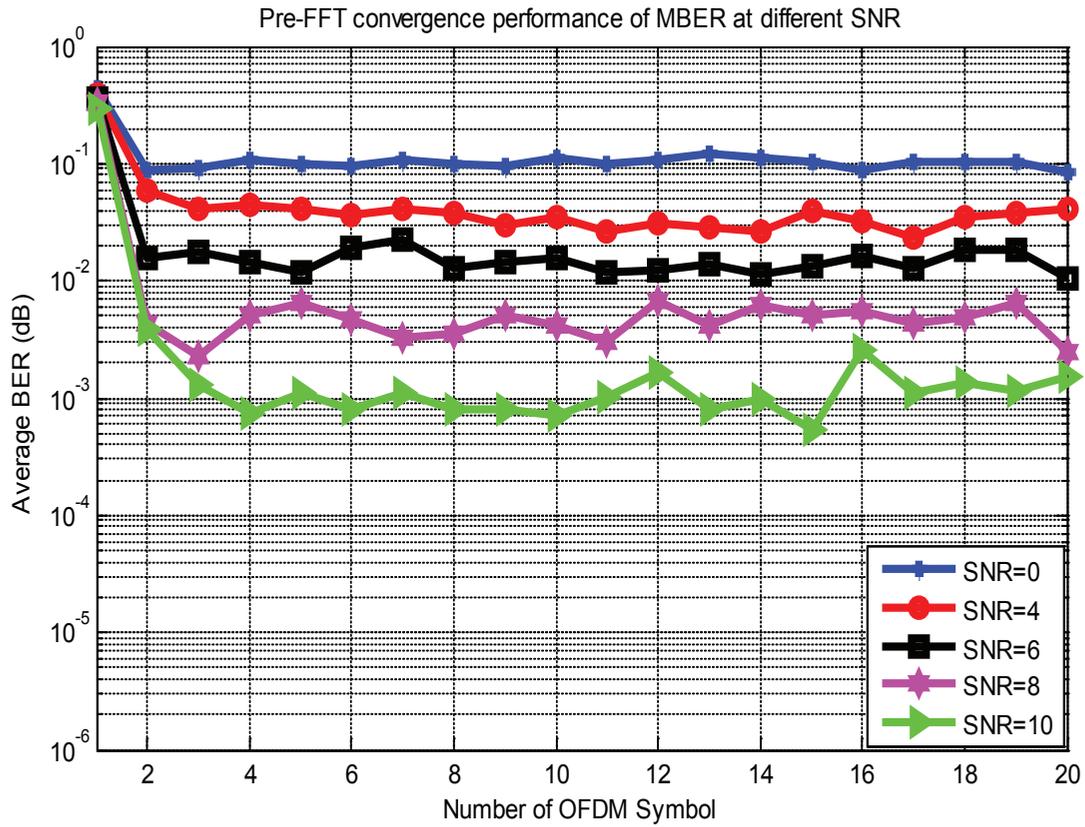


Fig. 9

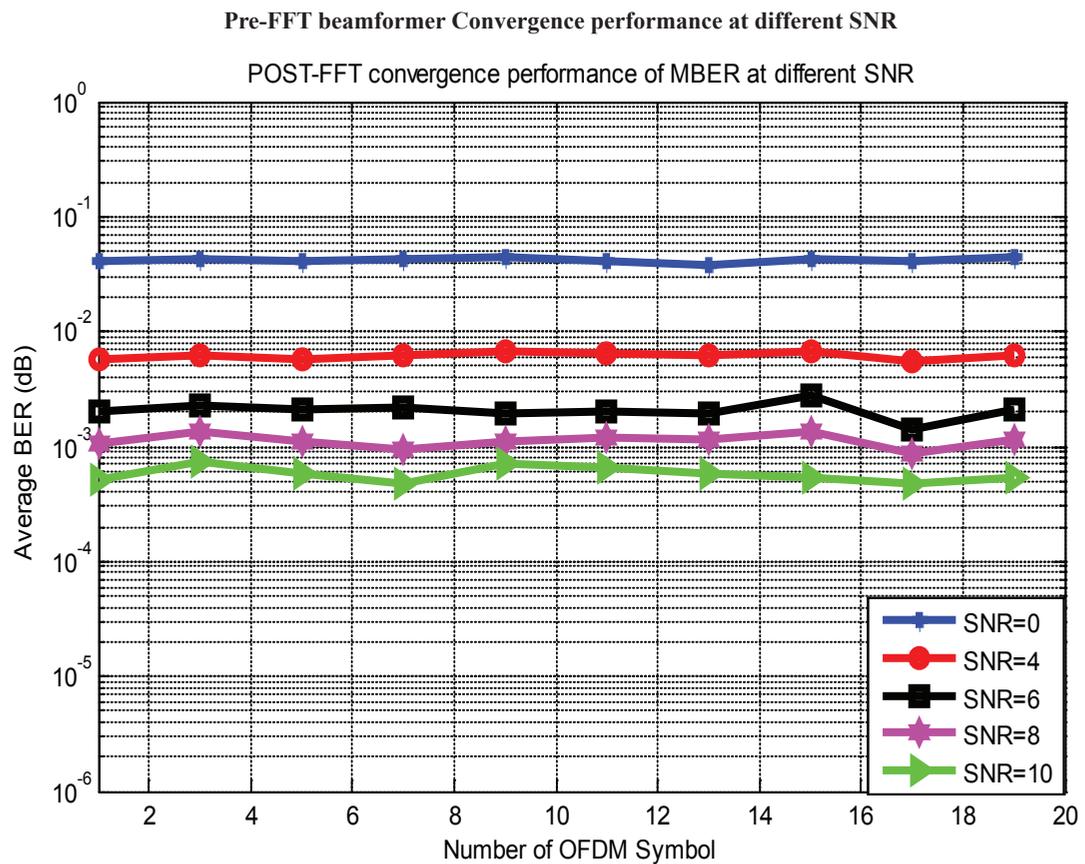


Fig. 10

Post-FFT beamformer Convergence performance at different SNR.

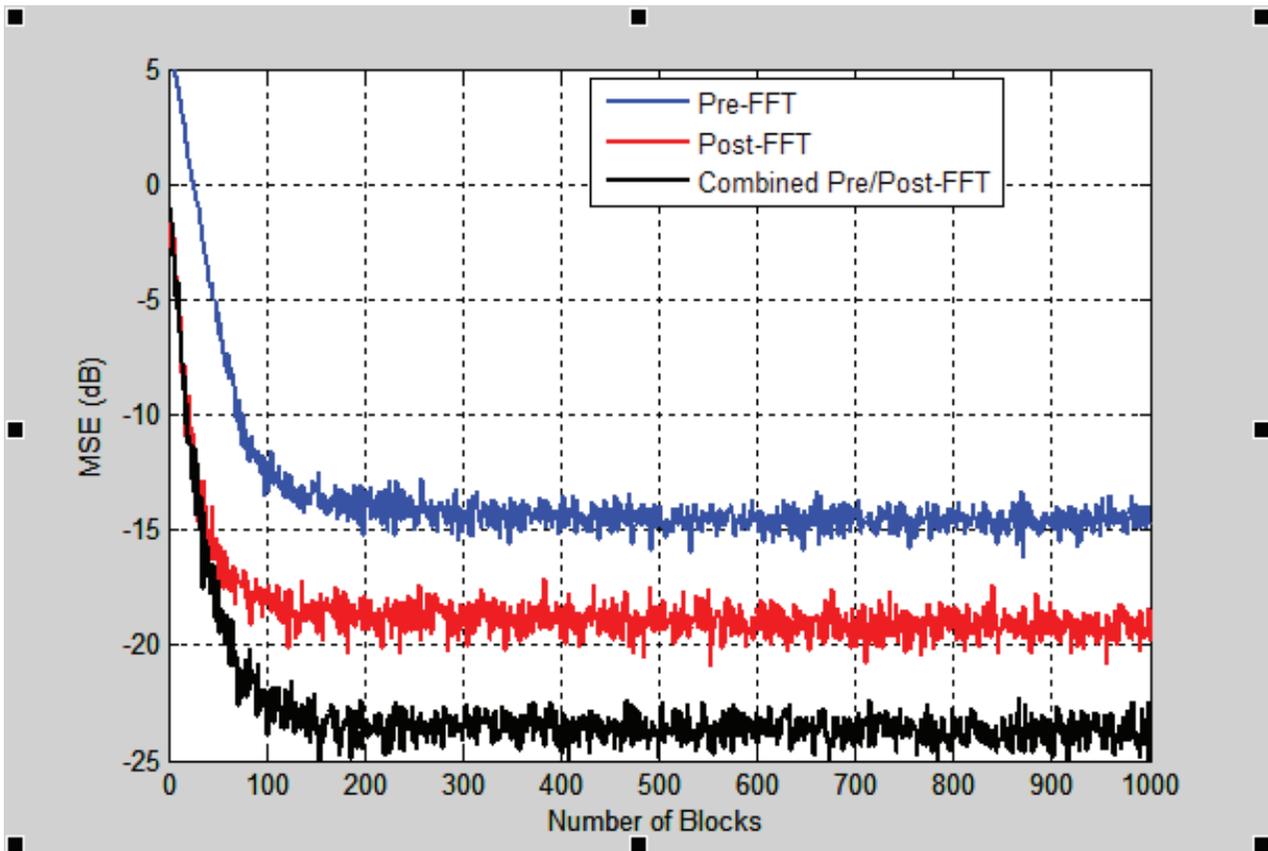


Fig. 11

Post-FFT/Pre-FFT beamformer for LMS Convergence performance at SNR=25dB.

Fig. 11 shows the LMS curves of Pre-FFT, Post-FFT and combined LMS scheme with SNR=25 dB. It is noted that the LMS curve of the Post-FFT scheme is better than the Pre-FFT scheme. The combined scheme provided much

better performance in case of BER compared to individual schemes at the cost of increased system complexity. However, the convergence rate for all schemes is almost similar.

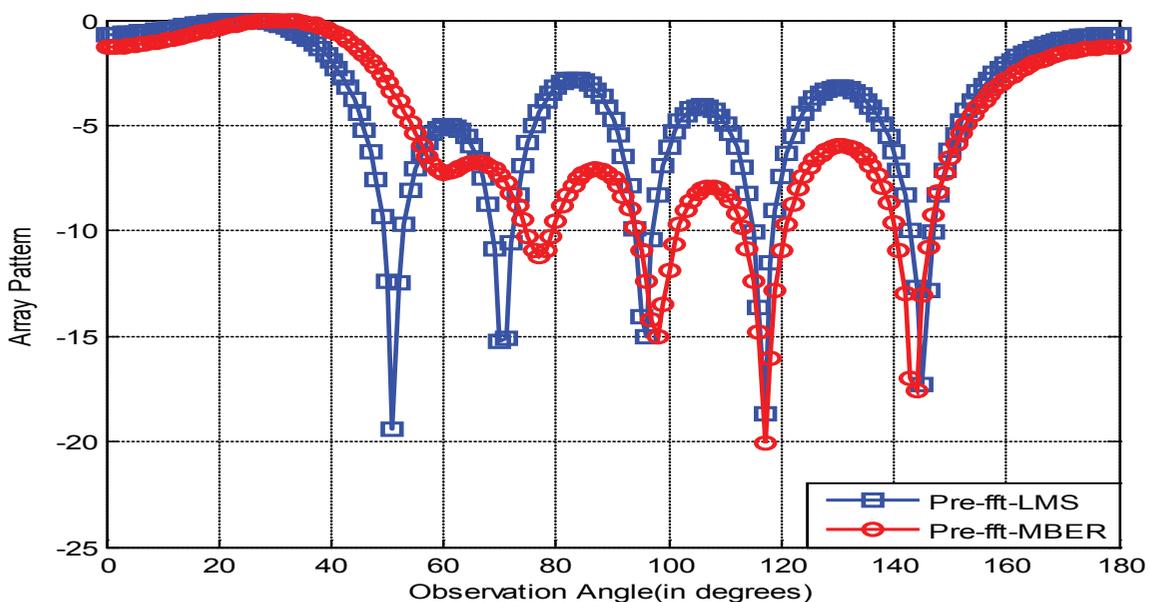


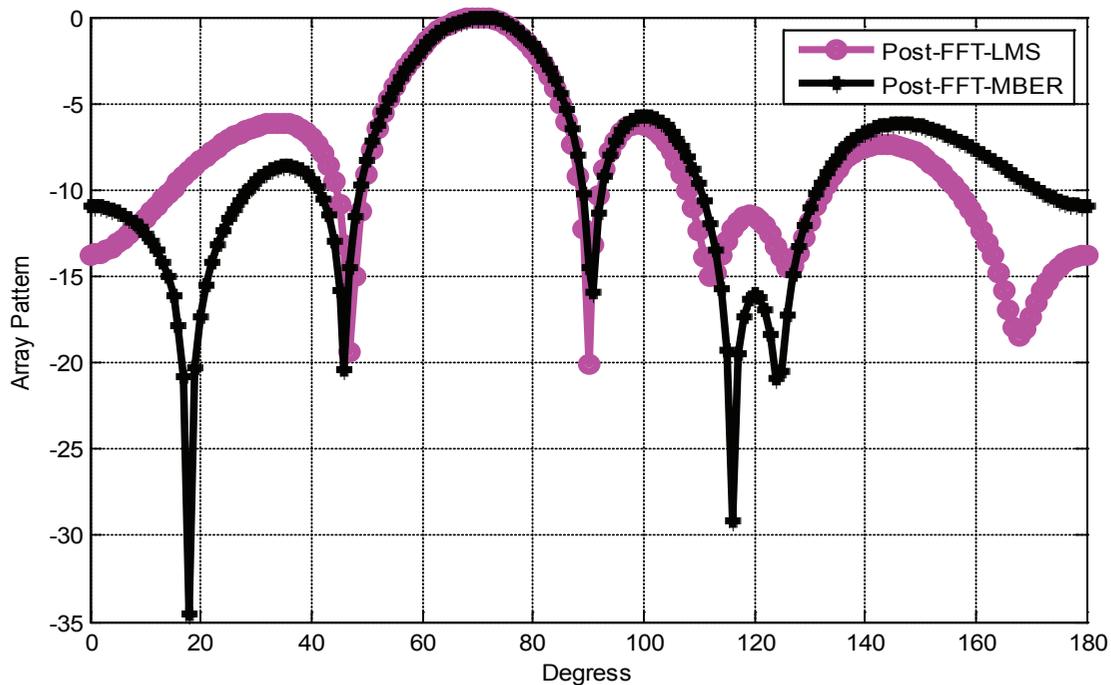
Fig.12.

Beampattern of the LMS and MBER beamformer using 6 antenna elements and 3 users with SIR= -3dB.

Fig.12 illustrates the beam pattern of the MBER and LMS beamformers for Pre-FFT OFDM adaptive antenna array. It shows that the MBER Pre-FFT beamformer has lower sidelobe levels.

In the post-FFT method, as shown in Fig.13 different paths of an interference source are

weight such that their combination is canceled and a null is not necessarily required to indicate better performance. Other subcarriers also demonstrated similar behavior. When the antenna spacing is small, different paths are mostly correlated. It is shown that the MBER has also lower sidelobes compared to LMS method.



**Fig.13.**

Beam pattern of the LMS and MBER beamformer using 6 antenna elements and 3 users with SIR= -3dB.

## CONCLUSION

In this paper, we studied the performance of the MBER for both Pre-FFT and Post-FFT for an OFDM adaptive antenna array system. Results were compared with LMS algorithm applied in similar channel conditions. A multipath (frequency selective fading) channel model is considered. Simulation results showed that MBER beamformer has provided better BER performance with acceptable levels of complexity and shorter training symbols. Our results show that Post-FFT has better performance compared with Pre-FFT in terms of BER but it requires more computations resulting in a more complex system. Also, a combined Pre-FFT/Post-FFT scheme was used in case of LMS and MBER. Although the combined scheme added slightly more complexity to the system (compared to Post-FFT) but it was

able to provide a notable increase in performance and produced better results in all scenarios without adding much computational load. Moreover, the MBER combined scheme offered the best performance of all schemes because it has superior BER results with reasonable increase in complexity which makes it suitable for practical applications.

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