٢٢- مخروط بوظة يبلغ قطره ٨سم وارتفاعه ١٢سم، وعلى قمته كرة بوظة قطرها ٨سم إذا ذابت البوظة كليا، هل ستسيل خارج المخروط أم لا؟ وضح. ما هو ارتفاع المخروط اللازم لكي لا تسيل البوظة إلى الخارج ؟ علما بان حجم المخروط $\frac{1}{3}\pi r^2 h$ وحجم الكرة $\frac{4}{3}\pi r^3$.

٢٣- اذا تقاطعت القطعتان المستقيمتان AB و CD في نقطة X بحيث كان (AX) (XD) (CX) = (XB) اثبت ان المثلثين XDB و XAC متشابهين.

٢٤- اذا لم يستطع علماء الرياضيات اثبات صحة معلومة رياضية ما ، علل ماذا يمكن ان نستنتج عن المعلومة.

٢٥- لديك الجملتان الرياضيتان التاليتان:

الجملة ١: س تؤدي إلى ص وتكتب بالرموز $[\tilde{O} \leftarrow \acute{O}]$ وهي تكافئ $[\acute{O} \sim \leftarrow \tilde{O} \sim]$ وتقرأ عكس ص تؤدي إلى عكس س.

الجملة ٢: ع تؤدي إلى عكس ص وتكتب $[\tilde{O} \sim \leftarrow \acute{U}]$.

ماذا يمكن أن نستنتج من الجملتين ١ و ٢؟

انتهى

١٧- ستخدم المسلمات التالية استخداما سليما في إثبات ان " لأي نقطة في النظام ، يوجد خط لا يقع عليها"

المسلمات هي : المسلمة الأولى: يوجد خط واحد في النظام على الأقل.

المسلمة الثانية: يقع على أي نقطتين خط وحيد.

المسلمة الثالثة: يقع على أي خط ثلاثة نقاط على الأقل.

المسلمة الرابعة: لأي خط يوجد نقطة لا تقع عليه.

١٨- اذا أعطيت المثلث ABC وكان قائما في A، النقطة M على الوتر BC، MA قائم على الوتر، اذا كان طول BM=4، وطول MC=5 اوجد طول كل من BA، AC، AM

١٩- ABCD شكل رباعي، نصفت أضلاعه بالنقاط F ، G ، H ، E ثم وصلت تلك النقاط معا لتكون الشكل الرباعي EHGF

بين ان الشكل EHGF متوازي أضلاع. ما هو الشرط الذي يجب ان نضيفه للمعطيات ليكون الشكل EHGF مستطيلا؟

الاسئلة (٢٠-٢٥) (١٥ علامة/ ٣ علامات لكل سؤال)

٢٠- اقرأ الجمل التالية بعناية:

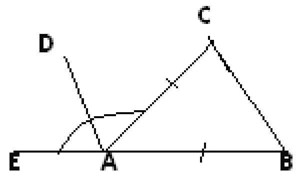
(١) إذا كان البعد بين أي مستقيمين ثابت ، فهما متوازيان.

(٢) المستقيمان المتعامدان على مستقيم ثالث متوازيان.

(٣) المستقيم المتعامد على أي من المستقيمين المتوازيين، يكون متعامدا على الآخر.

وضح كيف يمكن للجمل السابقة(كلها او بعضها) أن تكون سببا في أن مستقيما يوازي مستقيما آخر؟.

٢١- ABC مثلث فيه ‎$<A = ٣٠^{o}$‎ ، $AB = 10\sqrt{3}$ cm رسم BD عمودي على المستوى A،B،C بحيث BD=5cm ، رسم BM عمودي على AC ثم وصلت النقطة M مع D. اوجد قياس الزاوية الزوجية (B، AC، D)



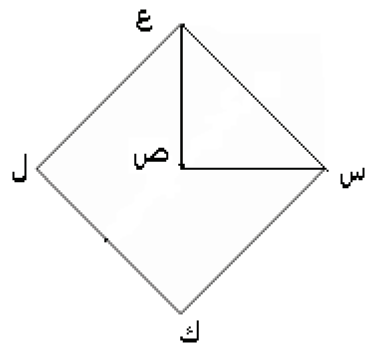
١٤- لديك النظرية التالية" قياس أي زاوية محيطية في دائرة واقعة على قوس يساوي نصف قياس الزاوية المركزية المرتبطة بالقوس المقابل للزاوية"
بالاستعانة بالنظرية السابقة بين أن الزاويتين المحيطيتين المرسومتين في دائرة ومشتركتان في نفس القوس متساويتان.

١٥ - مستعينا بسؤال(١٤) سابقا بين ان الزاوية المحيطية المقابلة لنصف قطر الدائرة تساوي قائمة.

الاسئلة من (١٦-٢٠) (١٥ علامة/ ٣ علامات لكل سؤال)

١٦- المثلث س ص ع قائم الزاوية في ص ومتساوي الساقين . تم إنشاء المربع س ك ل ع على وتر المثلث، كما في الشكل التالي:

يتبع ←

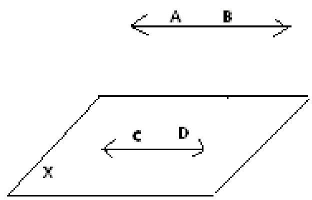


من هذه المعلومات يمكن أثبات أن مساحة المربع س ك ل ع هي أربعة أضعاف مساحة المثلث س ص ع.
هل يمكننا أن نستنتج من أن كل المثلثات القائمة والمتساوية الساقين تكون مساحة المربع س ك ل ع ، أربعة أضعاف مساحة المثلث س ص ع .

القسم الثاني

الاسئلة (١١-١٥) (١٥ علامة/ ٣ علامات لكل سؤال)

١١- أرتب المعطيات التالية لتكون برهانا للنتيجة التالية: "إذا وازى مستقيم AB خارج مستوىX ، مستقيما أخر CD يقع في المستوىX، فانه يوازي المستوى X.



المعطيات:

١- المستويان X،Y متقاطعان في المستقيم CD الواقع في كل منهما.

٢- وبالتالي لا يوجد نقطة مشتركة بين X و المستقيم AB .

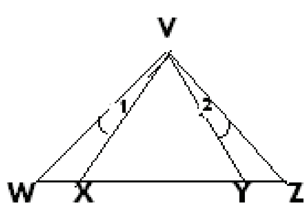
٣- لكن المستقيمان AB وCD متوازيان وغير متقاطعان.

٤- المستقيمان CD وAB محتويان في مستو واحد ليكن Y.

٥- إذا تقاطع المستقيم AB والمستوى X في نقطة فإنها تقع على CD.

٦- إذاهما متوازيان.

١٢- لديك المثلث WVZ متساوي الساقين فيه VZ=WV ، قياس <2=<1 ، بين أن المثلث XVY متساوى الساقين ؟



١٣- BAC مثلث متساوي الساقين فيه AC=AB ، EAC> زاوية خارجية للمثلث، نصفت الزاوية EAC> بواسطة القطعة المستقيمة AD ، برهن DA//CB.

٧- أي من التالية صحيحا
أ- المعين الذي إحدى زواياه قائمة هو مربع.
ب- قطرا المربع متساويان ومتعامدان.
ج- المعين الذي قطراه متساويان هو مربع .
د- جميع ما ذكر صحيحا.

٨- A،B مركزا دائرتين تتقاطعان عند النقطتان X،Y وينتج شكلا رباعيا AXBY أي الخيارات التالية صحيحا؟
أ- هناك ضلعان – على الأقل – متساويان في الشكل AXBY.
ب- المستقيمان AB و XY متعامدان.
ج- في الشكلAXBY زاويتان – على الأقل – متساويتان.
د- جميع ما ذكر صحيحا.

٩- دائرة مركزها C. AB قطر في الدائرة أي مما يلي يعد صحيحا دائما؟
أ- مماس الدائرة عند اية نقطة يكون عموديا على نصف قطر الدائرة AB.
ب- اذا تساوى طولا وتران في الدائرة فان بعديهما عن مركز الدائرة يكون متساويا.
ج- القطعة المستقيمة الواصلة بين مركز الدائرة ووتر فيها تعامد ذلك الوتر.
د- جميع ما ذكر صحيحا.

١٠- أي من التالية يعد صحيحا دائما في المجسمات ؟
أ- يمكن اعتبار المخروط القائم على انه هرما عدد أضلاعه كبير جدا .
ب- أي مقطع عرضي في أي هرم يكون مثلثا متساوي الأضلاع.
ج- يمكن تقسيم المعين الى أربعة مثلثات متطابقة.
د- لا شيء مما ذكر سابقا صحيحا.

يتبع ←

ملحق
اختبار مستوى التفكير الهندسي

الاسم:_______________ المنطقة: _______________

عدد سنوات الدراسة الجامعية:__________________

القسم الاول

(الاسئلة من (١-٥) اجب بنعم او لا (١٥ علامة/ ٣) علامات لكل سؤال:

١- لديك الجمل التالية: الجملة س: المثلث A ينطبق على المثلث B.
الجملة ص: المثلثB ينطبق على المثلث C.
الجملة ع: المثلث A ينطبق على المثلث C.

٢- اذا علم ان الجملة ع خاطئة فان إحدى الجملتين س أو ص تكون خاطئة.

٣- اذا قطع مستوى مستويين متوازيين فانه يقطعهما في مستقيمين متوازيين .

٤- ABCDشكل رباعي ، فيه قياس $<A$ + قياس $<B = 180^o$. فان الشكل يعتبر رباعيا دائريا.

٥- المثلث ABC فيه قياس $<A$ اكبر من قياس $<B$. فان طول الضلع BC اكبر من طول الضلع AC.

٦- المستقيم العمودي على احد مستويين متوازيين يكون عموديا على المستوي الثاني أيضا.

الاسئلة من (٦-١٠) اختر الإجابة الصحيحة (١٥ علامة/ ٣ علامات لكل سؤال):

٦- أي من التالية يعتبر صحيحا في المثلث ABC المتساوي الساقين:
ا- جميع زوايا المثلث متساوية. ب- جميع أضلاع المثلث متساوية.
ج- اذا كان قياس زاوية رأسه ضعفي قياس إحدى زوايا القاعدة، فان قياس زاوية الرأس يساوي ٤٥°.
د- العمود المقام من منتصف قاعدة المثلث ABC ينصف زاوية الرأس.

Knight, K.C. (2006). An Investigation into the Change in the Van Hiele Level of Understanding Geometry of Pre-service Elementary and Secondary Mathematics teachers. Unpublished Masters Thesis. University of Main.

Marzano,R. Pickering, J. Pollock,J.(2001) Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement Association for Supervision and Curriculum Development, ISBN: 0871205041 .

Van Hiele, P. M. (1986). Structure and Insight: A Theory of Mathematics Education. New York: Academic Press.

Yazdani, M. (2007). Correlation between students' level of understanding geometry according to the Van Hieles' model and students achievement in Plain geometry, Journal of Mathematical Sciences & Mathematics Education, February 2007,Vol. 2, No. 2

REFERENCE:

Burger, W. F., and Shaughnessy, J. M. (1986). Characterizing the Van Hiele levels of development in geometry. Journal for Research in Mathematics Education, 17, 31–48.

Chan, H., Tsai, P., and Huang, T.-Y. (2006). Web-based learning in a geometry Course. Educational Technology and Society, 9(2), 133-140.

Clements, D., and Battista, M. (1990). The effects of logo on children's conceptualizations of angle and polygons. Journal for Research in Mathematics Education, 21(5), 356-371.

Crowley, M. (1987). The van Hiele model of development of geometric thought. In M. M. Lindquist, (Ed.), Learning and teaching geometry, K-12 (pp.1–16). Reston, VA: National Council of Teachers of Mathematics.

Fuys, D., Geddes, D., and Tischler, R. (1988). The Van Hiele model of thinking in geometry among adolescents. Journal for Research in Mathematics Education Monograph No. 3. Reston, VA: National Council of Teachers of Mathematics.

Geddes, D., and Fortunato, I. (1993). Geometry: Research and classroom activities. In D. T. Owens (Ed.), Research Ideas for the Classroom: Middle grades mathematics (pp. 199–225). New York: Macmillan Publishing.

Glassgown,N. Farrell, T. (2007). What Successful literacy Teachers Do? Research-Based Teachers. Corwin Press Logo.

Halat,E. (2006). Sex-related differences in the acquisition of the van Hiele levels and motivation in learning geometry. Asia Pacific Education Review, vol 7(2), 173-183.

Halat,E. (2007). Reform-based curriculum and acquisition of the levels. Eurasia Journal of Mathematics, Science and Technology Education. vol.3(1):41-49.

Halat,E. (2008). In-Service Middle and High School Mathematics Teachers: Geometric Reasoning Stages and Gender. The Mathematics Educator. 2008, Vol. 18, No. 1, 8–

Halat,E. (2008). Pre-Service elementary school and secondary Mathematics teachers' Van Hiele Levels and Gender Differences. IUMPST: The Journal. Vol 1 (Content Knowledge), May 2008. [www.k-12prep.math.ttu.edu]

at which college learners should be (Fuys et al. 1988). From this study we concluded that most of (QOU) math learners were performing at lower (VH) level than they should be.

The post-test and the pre-test results for the 18 learners indicated that teaching a course in geometry applying Research-Based Strategies significantly improved their (VH) levels. Such transition was also evident in the learners' reflections they produced while they were approaching the geometric problems in which they first formulated questions then made a conjecture about the possible outcomes, and then tried to justify their conjecture based on their explorations.

The findings of this study offer a variety of recommendations for curriculum planners and developers, educators, supervisors and policy makers to take into consideration the importance of Research-Based Strategies while designing and developing the geometric syllabus.

For further research: it is preferable to conduct a replica study on a larger sample.

Table 4:

Descriptive statistics and the dependent samples T-Test for the pre-test and the post-test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre-test	2.6667	18	1.53393	.36155
	Post-test	3.2778	18	1.07406	.25316

Paired Samples Test

	Paired Differences					T	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Pair 1 Pre-test post-test	.61111	.84984	.20031	.18850	1.03372	3.051	17	.007

P=0.05

The Null and Alternate Hypotheses were:

- H0 : μPre -μPost = 0
- H1 : μPost –μPre > 0

Table 4 indicates that the post-test mean of (3.2778) is greater than that of the pre-test mean of (2.6667). The mean score difference in terms of reasoning stages is statistically significant [t = 3.051), p = 0.007< 0.05].

DISCUSSION and CONCLUSION:

This research showed that most (QOU) mathematics learners fell within level II (Analysis) or within level III (Ordering). A small portion of the learners fell within level IV (Deduction) and level V (Rigor), the level

Table 3:

Frequency Table of (QOU) Math Learners' (VH) Geometric Thinking Levels

Level-V (Rigor)	Level-IV (Deduction)	Level-III (Ordering)	Level-II (Analysis)	Level-I (Visualization)	level-0 (Pre-recognition)	Level
9.6	19.8	25.7	28.5	14.5	1.9	%
29.4%		70.6%				

The Table indicated that most (QOU) math education learners attained at levels- II (Analysis) (28.5%) and level III (Ordering) (25.7%). This is in line with the findings of Knight (2006).

To answer the second question "Is there a difference between the (VH) geometric thinking levels before and after applying the Research-Based Strategies?", the 18 learners took the pre-test before studying a course in geometry developed according to the Research–Based Strategies. The goal of the course which was taught for one semester was to engage the learners in activities of higher thinking skills. Learners were given a clear set of goals, as well as a repertoire of learning strategies to help them process, remember, and express ideas about the material they were exposed to in the course. They were asked to observe access information, evaluate content in reference books or in the internet for credibility and infer consequences and possible meanings. Participants discovered various facts, relationships, structures or models for themselves. After the completion of the course the 18 learners were re-evaluated with a post-test.

The means of each the pre-test and post-test were calculated, a T-Test for depending samples was conducted on the data to determine if there was a statistical significance between the mean of the (VH) geometric thinking levels before and after the experiment. Table 4 shows the descriptive statistics and dependent sample T-Test, for the pre-test and the post-test for the 18 learners.

Question:	1	2	3	4	5	overall
Reliability:	0.81	0.64	0.67	0.66	0.62	0.76

Table 2 - Kuder-Richardson inter-term reliability coefficient

The overall Kuder-Richardson indicated that the test is reliable.

DATA and **ANALYSIS:**

To answer the first question of the study “What are the (VH) levels of geometric thinking of (QOU) math education learners?” the 207 participants took geometry test. The test was graded according to the following criteria:

- The learner was classified in the first level if he/she answered 60% or more of the first level questions correctly.
- The learner was classified in the second level, if he/she answered 60% or more of the second level questions and met the criteria of the first level; and answered correctly less than 60% of levels III, IV and V questions.
- Therefore, the participants (VH) level of geometric thinking was determined according to the successfully answered questions (60% or more at and below that level).

A learner was given a score in the following way:

1 point for meeting criterion on level-I

2 points for meeting criterion on level-II

3 points for meeting criterion on level-III

4 points for meeting criterion on level-IV

5 points for meeting criterion on level-V

The percentages of each level were calculated. The results showed that 70.6% of the 207 learners fall in the third level (Ordering) of (VH) geometric thinking levels or below, while 29.4% only were either in the fourth (Deduction) level or fifth (Rigor) level. Table 1 shows the (QOU) mathematics learners’ (VH) levels distribution.

METHODOLOGY:

Participants

The study took place at Alquds Open University in Palestine. Two samples were selected

1. To investigate the (VH) levels of geometric thinking a random sample of 207 mathematics educational (QOU) learners were chosen.
2. To study the effect of Research-Based Strategies on the learners (VH) geometric level of thinking a convenient sample of 18 mathematics educational (QOU) learners were chosen.

Test Tool

A geometrical test tool was designed according to (VH) geometric thinking levels. It was used as the pre- and post-test. The test consisted of two parts; part one covered levels I and II of (VH) levels. It is composed of objective questions (true-false, multiple choice questions). Part two covered levels III, IV, V it is composed of subjective questions. The questions were arranged as follows.

Table 1:
Types of questions corresponding to (VH) levels

Question	Level	Question Type
1-5	I	True-false
6-10	II	Multiple choice
11-25	III,IV,V	Subjective

(See Appendix A)

Validity of the test

The test tools were validated by experts from the faculty of Mathematics Education Department at (QOU).

Reliability of the test

The reliability of the test was measured using a pioneer sample of 30 (QOU) math learners who were randomly selected to take the test. Table 2 below shows the Kuder-Richardson inter-term reliability for each question.

6. Cooperative learning: This strategy provides learners with opportunities to interact with each other in variety of ways.
7. Setting objectives and providing feedback: Setting objectives establishes a direction for learning. Once learners understood the parameters of an objective, they should brainstorm to determine what they know and what they want to learn.
8. Generating and testing hypotheses: The strategy of generating and testing hypotheses includes several processes such as; system analysis, invention, experimental inquiry, decision making and problem solving.
9. Questions, cues, and advance organizers: This strategy gives learners a preview of what they are about to learn or experience, it helps them activate prior knowledge; also provide them with the opportunity to connect what they already know to what they need to know.

Glasgow, N. Farrell, T (2007) stated that Research–Based Strategies are powerful tools engaging; guiding and monitoring learners' progress in participatory learning and can be more effective than traditional Classroom-Based Instructions.

QUESTIONS:

The objectives of the study were to answer the following questions:

1. What are (VH) levels of geometric thinking of (QOU) math education learners?
2. Is there a difference between the (VH) geometric thinking levels of (QOU) math education learners before and after applying the Research–Based Strategies?

SIGNIFICIANCE of the STUDY:

The effect of Research-Based Strategies on (VH) geometric thinking levels hasn't been tested in Palestine - to the researcher knowledge- accordingly, this study could benefit the educators, curricula planers and designers in the Ministry of Education in Palestine.

(1986), along with Geddes and Fortunato (1993), Crowley, (1987) and Fuys et al. (1988), argued that the quality of instructions had the greatest influence on the learners' acquisition of geometric knowledge in mathematics classes that affected their progress from one reasoning (VH) level to the next.

The Van Hiele theory indicates that effective learning that leads to raising (VH) levels, takes place when learners actively experience the objectives of study in appropriate contexts, and when they engage in discussion and reflection, thus lecturing and memorization as main methods of instruction will not lead to effective learning Chan, Huang (2006).

Marzano, Pickering, and Pollock (2001), in their book" Classroom Instruction that Works" utilized meta-analysis (a statistical technique) to analyze and summarize thousands of research studies that connect research recommendations to practice. They identified nine teaching and learning strategies that improve effective learning and learners' achievements. These key Research-Based Strategies are organized into categories as follows (Marzano, Pickering, and Pollock: 2001. p 13-111):

1. Identify similarities and differences: Identifying similarities and differences can be accomplished in a variety of ways like comparing, classifying, creating metaphors or creating analogies.
2. Summarizing and note taking: Summarizing and note taking can be done by deleting trivial material that is unnecessary, substitute terms for lists or select a topic sentence or invent one if it is missing.
3. Reinforce effort and provide recognition: This strategy addresses learners' attitudes and beliefs. When learners are rewarded or praised for achieving specific goals, their levels of achievement increases.
4. Homework and Practice: It provides opportunities for learners to practice, review, and apply knowledge. It also enhances a learner's ability to reach the level of expected proficiency for a skill or a concept. Marzano, Pickering, and Pollock's in their book indicated that learners need to practice a skill 24 times to reach 80% competency, noting that the first four practices yield the greatest effect.
5. Nonlinguistic representations: This strategy can enhance learners' ability to represent and to elaborate on knowledge using their own mental images.

INTRODUCTION:

Pierre M. Van Hiele, along with his wife Dina M. Van Hiele, developed a learning theory for geometry. The Van Hiele (VH) theory sets forth a learning model in which learners pass through five different sequential and hierarchical levels of thinking as they develop from a holistic understanding of geometric figures to an understanding of formal deductive geometric proof. Van Hiele suggested that the learners passed through several levels of reasoning about geometric concepts.

Yazdani (2007, p. 40) and (Chan et al., 2006) stated the following levels Van Hieles' model:

1. Level I (Visualization): learners identify shapes according to appearance.
2. Level II (Analysis): Learners reason geometric concepts by means of an informal analysis of component parts and attributes.
3. Level III (Ordering): Learners order properties logically and begin to appreciate the role of general definitions. In this level, learners can also form abstract definitions and distinguish between the necessity and sufficiency of a set of properties in determining a concept.
4. Level IV (Deduction): The role of axioms, undefined terms, and theorems are fully understood, and original proofs can be constructed.
5. Level V (Rigor): Learners can compare various axiomatic systems based on various axioms, and study various geometries in the absence of concrete models.

Many studies were performed to determine (VH) reasoning levels for middle and high school learners, and college learners in geometry. For example, Burger and Shaughnessy (1986) and Halat (2006, 2007) have found that grade k-8 learners are in level I (Visualization). By the end of the 8th grade, learners should be able to perform at level II (Analysis), and by the end of the12th grade learners should be able to perform at level III (Ordering) or level IV (Deduction). Fuys et al (1988) and knigh (2006) agreed that level V (Rigor) is more appropriate for college learners.

Halat (2008) claimed that there were many factors, such as gender, peer support, age, type of mathematics courses, instructions, etc… appear to be affecting pre-service mathematics teachers or college learners' performance and motivation in mathematics. While Mayberry, Burger and Shaughnessy

Abstract:

Van Hiele (VH) levels of geometric thinking were investigated on a sample of 207 of mathematics education learners at Alquds Open University (QOU). The results showed that 70.6% of them were within the third level of (VH) geometric thinking or below, while 29.4% were within the fourth and the fifth levels. This indicates that most of (QOU) math learners were performing at a lower (VH) level than they should be.

The second objective of this study was to determine the effect of Research-Based Strategies of teaching and learning on raising the (VH) geometric thinking. A group of 18 (QOU) mathematics education learners studied a geometric course according to Marzano's Research-Based Strategies. The results of the learners improved significantly which indicated the effectiveness of employing these strategies in raising learners' thinking levels of (VH).

Accordingly, it is recommended that curriculum planners and developers, educators, supervisors and policy makers take the Research-Based Strategies into consideration when designing and developing the syllabus.

ملخص:

بحثت هذه الدراسة في مستويات التفكير الهندسي لفان هيل على عينة حجمها ٢٠٧ دارسين من دارسي التربية تخصص الرياضيات في جامعة القدس المفتوحة، وأظهرت النتائج أن ٧٠,٦٪ منهم يقعون في المستوى الثالث من مستويات التفكير الهندسي لفان هيل أو دونه، وأن ٢٩,٤٪ يقعون إما في المستوى الرابع أو الخامس. وهي نتائج تبين تدني مستويات التفكير لديهم .

والهدف الثاني لهذه الدراسة هو تقصي مدى فاعلية استراتيجيات التعلم بالبحث في رفع مستويات التفكير الهندسي لدى الدارسين، حيث درست عينة تكونت من ١٨ دارسا من دارسي التربية تخصص الرياضيات مقرراً هندسياً صمم وفق استراتيجيات مارزانو للتعلم بالبحث. وأظهرت النتائج فاعلية استخدام هذه الإستراتيجيات في رفع مستويات فان هيل لتفكير الهندسي لديهم.

وأوصت الدراسة بضرورة قيام مخططي ومطوري البرامج الدراسية وصانعي السياسات التربوية بتوفير المحتوى الدراسي الذي يأخذ استراتيجيات التعلم بالبحث بعين الاعتبار.

The Effects of Research-Based Strategies on Raising the Van Hiele Levels of Geometric Thinking

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