Minimum Bit Error Rate Pre-FFT Beamforming for OFDM Communication Systems*

Waleed Abdallah**

* Received: 11/12/2012, Accepted: 4/5/2013.
** Assistant Prof., Technology and Applied Sciences Collage, Al-Quds Open University, Ramallah & Al-Bireh Branch, Palestine.

ملخص:

نستخدم في الاتصالات اللاسلكية ذات النطاق الواسع، مصفوفة التكيف الهوائي، مع تقسيم الترددات المتعامدة، فباستخدامها نستطيع التغلب والتخفيف من وطأة التداخلات الناتجة عن المستخدمين من التردد الواحد نفسه، وكذلك التداخلات ما بين الرموز، وهناك طريقتان رئيستان تستخدمان مع تقسيم الترددات المتعامده وقد تم التركيز في هذا البحث على نوع واحد وهو Pre-FFT بحيث يُستخدم لحساب الوزن في مجال الزمن الذي يعمل على تشكيل الأشعة وتركيزها باتجاه الإشارة المرغوبة فيها وإزالة التداخلات الناتجة من اتجاهات عدة.

وقد اُستخدمت خوارزمية جديدة وطُوِّرت كي تعمل عل تقليل معدل الخطأ إلى أدنى مستوى ما بين قوة الإشارة إلى قوة الضوضاء، وقد تبين لنا أن هذه الخوارزمية كانت أقل تعقيداً، وأحسن أداء مقارنة بالنظم السابقة التي اُستخدمت حيث دُرست الخوارزمية السابقة المذكورة فيه وقورنت بالخوارزمية المقترحة في هذا البحث، حيث تبين بأن الخوارزمية الجديدة المقترحة أفضل في الأداء وأقل تعقيداً وقدرتها على تخفيف وطأة تعدد المسارات الناتجة عن بث الإشارات والتداخلات الناتجة عن المستخدمين للتردد الواحد.

Abstract:

In broadband wireless communication, the adaptive antenna array (AAA) is combined with orthogonal frequency division multiplexing (OFDM) to combat the inter-symbol interference (ISI) and the directional interferences. One of the two main techniques which are used in OFDM systems is called Pre-FFT, where an optimum beamformer weight set is obtained in time domain before Fast Fourier Transform (Pre-FFT). In this paper, the optimum weight set is obtained based on minimum bit error rate (MBER) criteria in pilot-assisted OFDM systems under multipath fading channel. Here, the development of the block-data adaptive to implementation of the MBR beamforming solution is based on the Parzen window estimate of probability density function (pdf). The simulation results show that the MBER technique utilizes the antenna array elements more intelligently than the standard minimum mean square error (MMSE) technique. The MBER technique can be used to achieve excellent performance since it directly minimizes the BER and requires shorter training symbols.

Index Terms: MBER beamforming, OFDM systems, Pre-FFT, MMSE beamforming, probability density function (pdf), smart antenna.

I. Introduction:

It is well known that the orthogonal frequency division multiplexing (OFDM) can be considered as an efficient technique for high speed digital transmission over severe multipath fading channels where the delay spread is larger than the symbol duration. When the inserting guard time is longer than the delay spread of the channel this makes the system robust against inter-symbol interference (ISI). In addition to that, channel estimation and compensation can be achieved by inserting known pilot symbols between data symbols [1]-[3].

Over the last few years, adaptive antenna array (AAA) has gained much attention due to its ability to increase the performance of wireless communication system, in terms of spectrum efficiency, network scalability, and operation reliability. Adaptive beamforming can separate signals transmitted in the same carrier frequency, provided that they are separated in the spatial domain. The beamformer combines the signal received by the different element of an antenna array to form a single output. The adapted weight set of each element of the antenna array is obtained by the processor achieving certain criteria to suppress the co-channel interference; thus improving coverage quality.

The main motivation behind Pre-FFT scheme is reducing the cost due to FFT processing [1, 7]. The weight obtained for each pilot subcarrier can be identically applied on all data subcarriers in the same OFDM symbol; thereby reducing the number of frequency domain narrow-band beamformers. Post FFT is not always better in performance than pre-FFT [1]. In[3], a pre-FFT least mean square (LMS) beamforming for OFDM systems was analyzed in additive Gaussian noise channel. An adaptive MBER beamforming was analyzed in [4] for single carrier modulation and in [2] for OFDM systems in additive Gaussian noise channel. A class of MBER algorithms were studied in [5] and combined with space time coding in [6]. Eigenvector combining was considered in [8]. MIMO MBER beamforming for OFDM was studied in [9]. A block-by block post-FFT multistage beamforming was considered in [10].

The MMSE technique does not guarantee the minimum of the

BER, but the MBER technique is characterized by its good performance and amenability to adapt on implementation. In [1] and [2] the MMSE and

MBER beamformers for Pre-FFT OFDM are presented, respectively, without investigating several factors affecting performance. The channel is assumed to be non-dispersive with additive Gaussian noise, which is not a practical channel. Since new wireless standards, such as IEEE 802.11 and 802.16, use the pilot subcarriers in their structures, our focus in this paper will be given to suppress co-channel interference and mitigate the multipath interference in pilot-assisted OFDM systems. In [3], the MMSE beamforming algorithm for Pre-FFT OFDM system is applied on a channel assumed to be frequency selective fading.

The main contribution in this paper is to analyze the MBER algorithm in a practical channel model which is assumed to be

multipath frequency selective fading channel as described in [3]. Comparative studies are conducted between the MMSE algorithm, to [3], and the proposed MBER algorithm in terms of several factors: presence of multiuser interference with different powers; the number of antenna elements; the number of paths and the signal separation (angle difference between the desired and the interferences).

Simulation results showed that the MBER algorithm structure had the lowest computational complexity, and the best BER performance than MMSE algorithm.

This paper is organized as follow: Section II describes the Pre-

FFT OFDM system model. In Section III, Pre-FFT adaptive beamforming to based on MBER criteria is introduced. Section IV provides simulation results and comparative studies. Finally, conclusions and possible directions for future work are presented in section V.

II. PRE-FFT OFDM System model:

If we consider that M-user OFDM system uses K subcarriers for parallel transmission, [2]. The sample modulated by the kth subcarrier of the mth user is given by

$$x_m(k) = b_m(k) \qquad 1 \le m \le M \qquad 1 \le k \le K \quad (1)$$

where $b_m(k) \in \{\pm 1\}$ for BPSK signals. We assume that user 1 is the desired user and the other sources are interfering users. This data can be interpreted to be a frequency-domain data and subsequently converted to a time-domain signal by an IFFT operation. This process can be written as,

$$\bar{y}_m = \frac{1}{K} F^H \bar{x}_m \qquad 1 \le m \le M \quad (2)$$

where

$$\bar{y}_m = [y_m(1), y_m(2), \ldots, y_m(K)]^T \quad (3)$$

$$F = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{\frac{-j2\pi(1)(1)}{K}} & \cdots & e^{-\frac{j2\pi(1)(K-1)}{K}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{\frac{-j2\pi(K-1)(1)}{K}} & \cdots & e^{-\frac{j2\pi(K-1)(K-1)}{K}} \end{bmatrix} \qquad (4)$$

representing the FFT operation matrix,

$$\bar{x}_m = [x_m(1), x_m(2), \ldots, x_m(K)]^T \qquad (5)$$

and H denotes the Hermitian transpose of a matrix. The output of the IFFT is transmitted to the channel after the addition of cyclic prefix (CP). In order to add the CP, $\bar{y}_m$ is cyclically extended to generating $\tilde{y}_m$ by inserting the last v element of y_m at its beginning, i.e.

$$\tilde{y}_m = \begin{bmatrix} J_v \\ I_K \end{bmatrix} \bar{y}_m \quad (6)$$

where J_v contains the last v rows of a size K identity matrix I_K.

Finally, the OFDM time signal is transformed to the analog form through D/A converter before transmission in the wireless channel. A multipath channel model (frequency selective fading) is assumed to include maximum of L paths and is assumed that the m^{th} source (desired or interference) and the receiving antenna array in the form of

$$h_m(k) = \sum_{l=0}^{L-1} \alpha_{m,l} \delta(k-l) \qquad m = 1, \cdots, M \quad (7)$$

where $\alpha_{m,l}$ denotes a complex random number representing the l^{th} channel coefficient for the m^{th} source and $\delta(.)$ is delta function.

Fig.1 illustrates the architecture of Pre-FFT beamforming at the receiver of an OFDM system. Assuming that the CP is longer than the channel length (v > L), the received signal on the nth antenna of a uniform linear array (ULA) for an OFDM block will be given as follow:

$$r_n(k) = \sum_{m=1}^{M}\sum_{l=0}^{L-1} \alpha_{m,l}\tilde{y}_m(k+v-l)e^{-j\frac{2\pi}{\lambda}(n-1)d\cos(\theta_{m,l})} + \eta_n(k) \quad (8)$$

$$1 \le n \le N \quad , \quad 1 \le k \le K$$

where N denotes the total number of antennas, λ represents the wavelength of the carrier, d denotes the inter-element spacing, and $\eta_n(k)$ represents the channel noise which enter the nth antenna. $\theta_{m,l}$ denotes the direction of arrival (DOA) of the lth path and m^{th} source. Without loss of generality, we assume here that the channels of all sources have the same length L.

At the receiver, the received signal with a spatial domain for nth array element is multiplied by the nth weight (w_n) of adaptive beamformer.

$$\overline{Z} = W^H \cdot \overline{R} \quad (9)$$

where

$$W = [w_1\, w_2 \quad \cdots w_N]^T \quad (10)$$

$$\overline{R} = [\overline{r}(1)\,\overline{r}(2) \quad \cdots \overline{r}(K)] \quad (11)$$

$$\overline{r}(k) = [r_1(k)\, r_2(k) \quad \cdots r_N(k)]^T \quad (12)$$

$$\overline{Z} = [z(1)\, z(2) \quad \cdots z(K)]^T \quad (13)$$

The sum of this signals ($\overline{Z}$) is transformed back into frequency-domain symbols ($\hat{Z}$) by applying the FFT operator. This process can be written as follows:

$$\hat{Z} = F \cdot \overline{Z} \quad (14)$$

where $\hat{Z}$ is the estimated frequency-domain symbols (data and pilot), and is given by

$$\hat{Z} = [\hat{z}(1)\ \hat{z}(2)\ \ldots, \hat{z}(K)]^T \quad (15)$$

and $\hat{z}(k)$ denotes the corresponding received sample at the kth subcarrier .

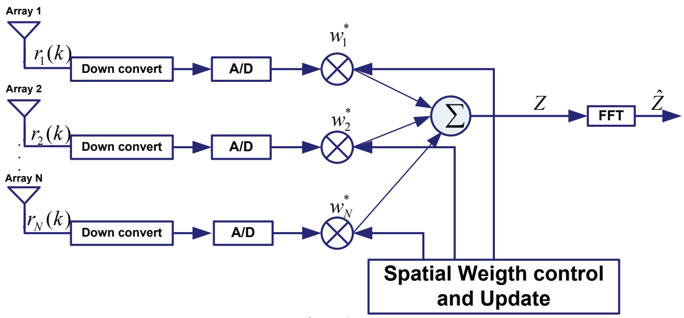

Fig.1.

Block diagram of the Pre-FFT OFDM adaptive receiver.

The estimate of the transmitted bit $b_1(k)$ is given by

$$\hat{b}_1(k) = \begin{cases} +1, & \mathrm{Re}(\hat{z}(k)) > 0 \\ -1, & \mathrm{Re}(\hat{z}(k)) \le 0 \end{cases} \quad (16)$$

where $\mathrm{Re}(\hat{z}(k))$ denotes the real part of $\hat{z}(k)$.

III. PRE-FFT OFDM SYSTEM BASED ON MBER CRITERIA:

In this section, Pre-FFT adaptive beamforming based on MBER criteria is introduced to obtain the optimum weight set. The theoretical MBER solution for the Pre-FFT OFDM beamformer is obtained in [2] where, the channel is assumed to be non-dispersive with additive Gaussian noise. The error probability (BER cost function) of the frequency domain signal of the beamformer is given by

$$P_E(W) = \mathrm{Prob}\{\mathrm{sgn}(b_1(k))\mathrm{Re}(\hat{z}(k)) < 0\} \quad (17)$$

where sgn(.) is the sign function. The weight vector that minimize the BER is then defined as

$$W = \arg\min_{W} P_E(W) \quad (18)$$

From equation (17), define the signed decision variable

$$\begin{aligned} \hat{z}_s(k) &= \mathrm{sgn}(b_1(k))\,\mathrm{Re}(\hat{z}(k)) \\ &= \mathrm{sgn}(b_1(k))\,\mathrm{Re}(\hat{z}'(k)) + \eta'(k) \end{aligned} \quad (19)$$

where

$$\hat{z}'(k) = W^H [\bar{r}(k) - \eta(k)] F(k) \qquad (20)$$

and

$$\eta'(k) = \text{sgn}(b_1(k)) \Re(W^H \eta(k) F(k)) \qquad (21)$$

$\hat{z}_s(k)$ is a very good error indicator for the binary decision, i.e., if it is positive, then the decision is correct, else if it is negative, then an error occurred, F(k) is the kth column of F. Notice that F is unitary matrix, so $\eta'(k)$ is still Gaussian with zero mean and variance $\sigma_n^2 \cdot W^H W$.

The conditional probability density function (pdf) given the channel coefficients $\alpha_{m,l}$ of the error indicator, $\hat{z}_s(k)$, is a mixed sum of Gaussian distributions [12], i.e.,

$$p_z(\hat{z}_s) = \frac{1}{K\sqrt{2\pi}\sigma_n\sqrt{W^H W}} \cdot \sum_{k=1}^{K} \exp\left(-\frac{(\hat{z}_s - \text{sgn}(b_1(k)) \Re(\hat{z}'_s(k)))^2}{2\sigma_n^2 W^H \cdot W}\right) \qquad (22)$$

and it is the best indicator of a beamformer's BER performance. Deriving a closed form for the average error probability is not easy.

Therefore, we use the gradient conditional error probability to update the weight vector.

The conditional error probability given the channel coefficients $\alpha_{m,l}$ of the beamformer, $P_E(W)$, is given by [4] :

$$\begin{aligned} P_E(W) &= \frac{1}{K\sqrt{2\pi}\sigma_n\sqrt{W^H W}} \cdot \sum_{k=1}^{K} \int_{q_k(W)}^{\infty} \exp\left(-\frac{u^2}{2}\right) du \\ &= \frac{1}{K} \sum_{k=1}^{K} Q(q_k(W)) \end{aligned} \qquad (23)$$

where

$$u = \frac{(\hat{z}_s - \text{sgn}(b_1(k)) \Re(\hat{z}'_s(k))}{\sigma_n \sqrt{(W^H W)}} \qquad (24)$$

where $Q(\cdot)$ is the Gaussain error function and is given by

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} \exp\left(-\frac{y^2}{2}\right) dy \qquad (25)$$

$$q_k(W) = \frac{\operatorname{sgn}(b_1(k))\,\Re(\hat{z}'(k))}{\sigma_n\sqrt{W^H W}} \quad (26)$$

In OFDM system as described in section II, it is assumed that in every symbol there are pilot signals in order to perform channel estimation. The pilot signals are also used in the adaptive update of the beamformer weight vector. So the transmitted pilot signal vector of the desired user, $\bar{x}_{1p}$, and the received pilot signal vector, $\bar{\hat{z}}_p$, in frequency domain can be written as follow [2]

$$\bar{x}_{1p} = [x_1(1), 0.., x_1(\Delta p+1), 0,.., x_1((K_p-1)\Delta p+1)), 0,..] \quad (27)$$

$$\begin{aligned}\bar{\hat{z}}_p &= [\hat{z}(1), 0.., \hat{z}(\Delta p+1), 0,.., \hat{z}((K_p-1)\Delta p+1), 0,..] \quad (28)\\ &= W^H \bar{R} F_p\end{aligned}$$

where $$F_p = \begin{bmatrix} 1 & 0 & \cdots & 1 & \cdots & 1 & 0\cdots \\ 1 & 0 & \cdots & e^{-j2\pi(1)(\Delta p)/K} & \cdots & e^{-j2\pi(1)(K_p-1)\Delta p/K} & 0\cdots \\ \vdots & 0 & & \vdots & \ddots & \vdots & 0\cdots \\ 1 & 0 & \cdots & e^{-j2\pi(K-1)(\Delta p)/K} & \cdots & e^{-j2\pi(K-1)(K_p-1)\Delta p/K} & 0\cdots \end{bmatrix} \quad (29)$$

representing FFT operation matrix at the pilot locations and Δp represents the frequency spacing between consecutive pilot symbols. The first pilot symbol is assumed to be positioned at the first subchannel. For each OFDM symbol, K_p pilot signals in the frequency domain are required to obtain the optimum weight vector.

The method of approximating a conditional pdf known as a kernel density or Parzen window-based estimate, [9], is used to estimate the conditional error probability given the channel coefficients $\alpha_{m,l}$ is used on OFDM systems. Given an OFDM symbol of K_p training samples $\{\bar{r}(k), b_1(k)\}$, a kernel density estimate of the conditional pdf given the channel coefficients $\alpha_{m,l}$ at pilot locations which was defined in (22), is given by

$$\begin{aligned}\hat{p}_z(\hat{z}_s) &= \frac{1}{K_p\sqrt{2\pi}\rho_n\sqrt{W^H W}} \times \\ &\sum_{k=0}^{K_p-1} \exp\left(-\frac{(\hat{z}_s - \operatorname{sgn}(b_1(k\times\Delta p+1))\,\Re(\hat{z}(k\times\Delta p+1)))^2}{2\rho_n^2 W^H W}\right)\end{aligned} \quad (30)$$

Therefore, the block cost function could be derived from the kernel density estimate of conditional pdf given the channel coefficients $\alpha_{m,l}$ as follows [4], [7], [12]:

$$\hat{P}_E(W) = \frac{1}{K_p}\sum_{k=0}^{K_p-1} Q(\hat{q}_k(W)) \quad (31)$$

where $$\hat{q}_k(W) = \frac{\mathrm{sgn}(b_1(k\times\Delta p+1))\mathrm{R}(W^H \overline{R} F_p(k\times\Delta p+1))}{\rho_n\sqrt{W^H W}} \quad (32)$$

and $F_p(k\times\Delta p+1)$ is the $(k\times\Delta p+1)^{th}$ column of F_p. From this estimated conditional pdf given the channel coefficients $\alpha_{m,l}$, the gradient of the estimated BER is given by [4]

$$\nabla\hat{P}_E(W) = -\frac{1}{\sqrt{2\pi}\rho_n\sqrt{W^H W}}\sum_{k=0}^{K_p-1}\exp\left(-\frac{(\mathrm{Re}(\hat{z}(k\times\Delta p+1)))^2}{2\rho_n^2 W^H W}\right) \quad (33)$$
$$\times\,\mathrm{sgn}(b_1(k\times\Delta p+1))\,\overline{R}F_p(k\times\Delta p+1)$$

Now a block-data adaptive MBER algorithm is obtained by the gradient of $\hat{P}_E(W)$. For each OFDM symbol, we can find the optimum weight vector W by the steepest-descent gradient algorithm [4]

$$\nabla\hat{P}_E(W) = -\frac{1}{\sqrt{2\pi}\rho_n}\exp\left(-\frac{(\mathrm{R}(\hat{z}(k\times\Delta p+1)))^2}{2\rho_n^2}\right) \quad (34)$$
$$\times\,\mathrm{sgn}(b_1(k\times\Delta p+1))\,\overline{R}F_p(k\times\Delta p+1) \qquad 1\le k\le K_p$$

That is to say, W weight vector can be updated K_p times in one OFDM symbol. Thus complexity is reduced and consequently, the update equation is given by

$$W(k+1) = W(k) - \mu\nabla\hat{P}_E(W)$$
$$= W(k) + \frac{\mu}{\sqrt{2\pi}\rho_n}\cdot\exp\left(-\frac{(\mathrm{R}(\hat{z}(k\times\Delta p+1)))^2}{2\rho_n^2}\right) \quad (35)$$
$$\times\,\mathrm{sgn}(b_1(k\times\Delta p+1))\,\overline{R}F_p(k\times\Delta p+1) \qquad 1\le k\le K_p$$

where μ is a step size.

The proposed MBER algorithm is summarized in Table I.

The proposed MBER algorithm is composed of two main loops. The outer loop is for each block of data and the inner loop is repeated over the same block of data until certain number of iterations reached.

In the main loop, we formulate a block of data (64 bits) from the output of the antenna array. In the inner loop the gradient vector is determined from (34) at pilot locations (Np =16). Then, we compute the weight update vector from (35). After the end of the inner loop, we determine the detected signal by multiplying the computed optimized weight vector with the received signal in order to use it in calculating the BER the last update at the end of each OFDM block $(W(K)$ which is used as an initial value in the next block. Then, we back to the main loop and form another block of data and so on. These processes iterate until we finish all the incoming data.

Table I.:

MBER algorithm summary.

Initialization
$i=1, \mu=.01$, Block size $K=64$ Calculate variance of noise σ_n Initial weight vector $W=.01 * ones(N,1)$
Outer loop (1: floor (all bits/Block))
Form a block of data from the received signals.
Inner loop (while $k<K_p$)
Calculate the gradient matrix over the block from equations (34). Update the weight matrix as $W(k)=W(k-1)+\mu(k)\nabla\hat{P}_E$ from equation (35). Normalize the solution $W(k+1)=W(k+1)/\lVert W(k+1)\rVert$
end of inner loop Determine the detected signals in order to be used for calculating the BER. Increment the block number $i=i+1$
end of outer loop

IV. SIMULATIONS RESULTS:

The computer simulations are performed for implementing the Pre-FFT LMS and MBER beamformer in 64 subcarriers (16 + 48). OFDM system perfectly synchronized, with a CP length larger than the channel length (v=16), BPSK modulation is used in the system with six ULA elements antenna and half-wavelength spacing. The example used in our computer simulation study considers one desired user with DOA of 0° and two interferers with SIR = -3dB. We further assumed normalized channels with different lengths and with real coefficient 0.864, 0.435, 0.253 and 0 for all sources, and an angle spread of $\pm 5^o$ (for all sources).

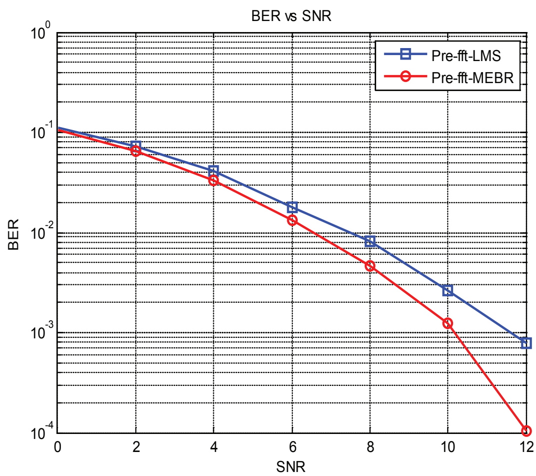

Fig. (2):

Comparison of the bit error performance when using 6 antenna elements and 3 users with SIR= -3 dB.

Fig.2 and Fig. 3 compare the BER performance of the MBER beamformer with that of the MMSE beamformer for SIR = -3 dB and 0 dB, respectively. In Fig. 2, the effect of changing the channel model on the proposed algorithm is illustrated. It is observed that the BER of the MBER beamformer is superior to that of LMS under moderate SNR.

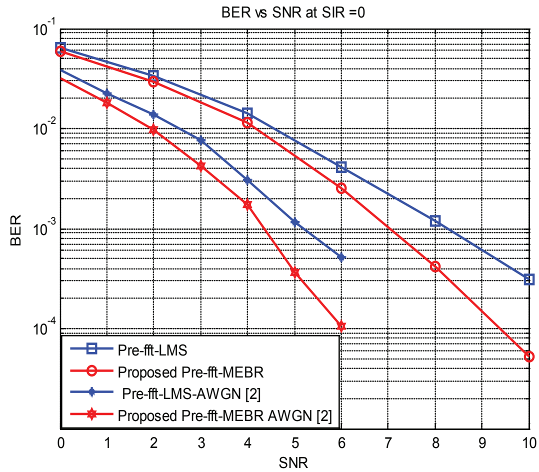

Fig. (3):

Comparison of the bit error performance when using 6 antenna elements and 3 users with SIR= 0 dB.

Fig.4 illustrates the beam pattern of the MBER and LMS beamformers for Pre-FFT OFDM adaptive antenna array. It shows that the MBER pre-FFT beamformer has lower sidelobe levels.

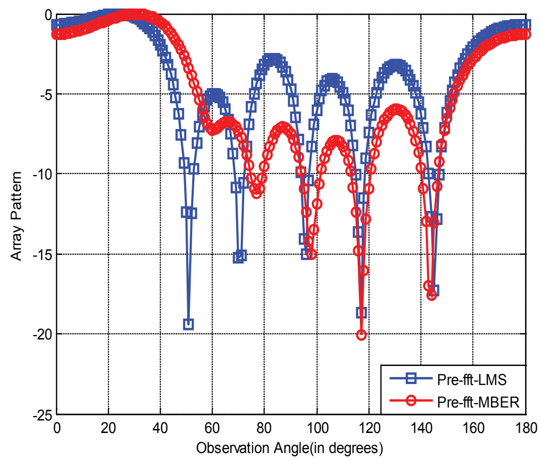

Fig. (4):

Beampattern of the LMS and MBER beamformer using 6 antenna elements and 3 users with SIR= -3dB.

Fig.5. illustrates the effect of the number of antennas. As it is expected, the BER performance increase with large number of antennas and this is because of high resolution of antenna beam, hence, a better control for the desired sources separation and interference rejection can be obtained in an

array. With more interference sources and delayed paths, the Pre-FFT scheme requires more antennas to put nulls at their angles.

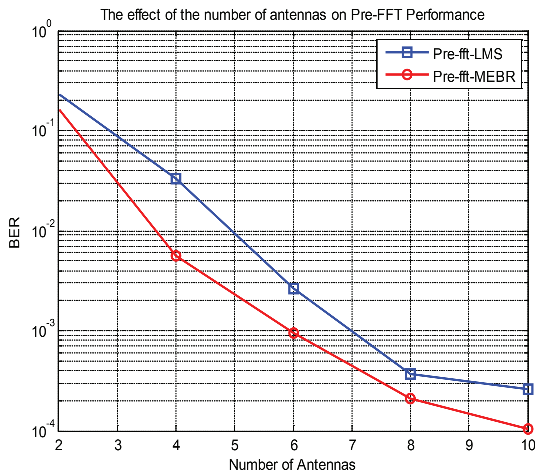

Fig. (5):

The effect of the number of antennas on the Pre-FFT performance.

Fig.6. illustrates the effect of the signal separation between the desired user and the two interferers. It is clearly seen that the Pre-FFT scheme encounters performance degradation when the signal separation decreases. The Pre-FFT scheme shows better results with wider angle separation.

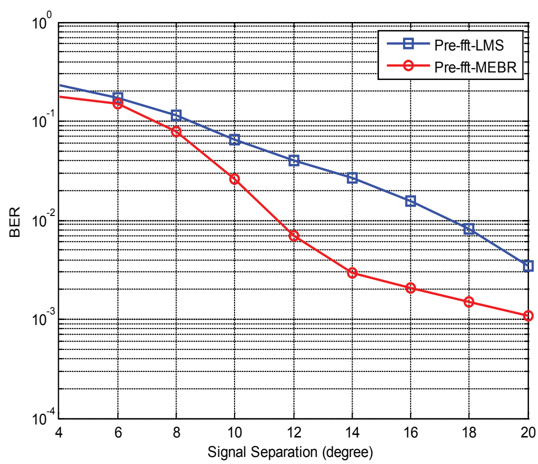

Fig. (6):

The effect of the angular separation between signals on the Pre-FFT performance.

Fig. 7 illustrates the effect of channel length, the number of paths were increased from three (coefficients 0.864, 0.435, 0.253), to five (coefficients 0.864, 0, 0.435, 0, 0.253), to seven (coefficients 0.864, 0, 0, 0.435, 0, 0, 0.253), and finally to nine (coefficients 0.864, 0, 0, 0, 0.435, 0, 0, 0, 0.253). The channels were kept normalized and their lengths remained less then the CP length to avoid ISI. Fig.7. shows the impact of channel length for the same system with 6 antennas, SNR=10dB, angle spread $\pm 15^{0}$ degrees. It is observed from the figure that the performance of the Pre-FFT scheme is not affected by the channel length for both MMSE and MBER techniques.

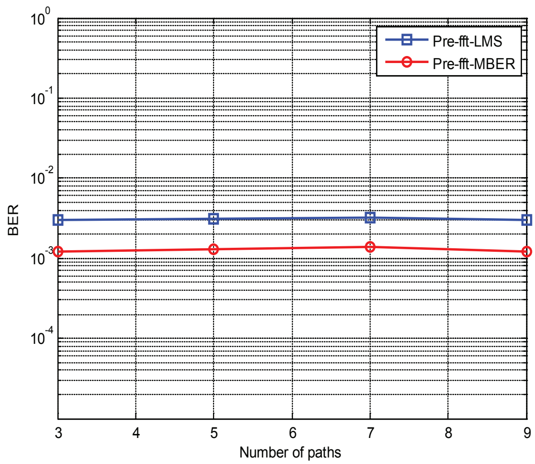

Fig. (7):

The effect of the channel length on the Pre-FFT performance.

Fig. 8 and Fig. 9 illustrate the convergence performance of MBER Pre-FFT beamformer. The Fig. 8 and Fig. 9, show that the optimum weights and the steady state BER performance, respectively, require about 5 OFDM symbols under the condition of SNR=10dB and step size $\mu = .01$.

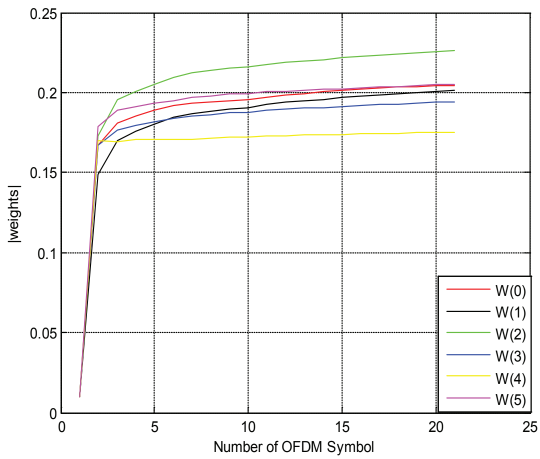

Fig. (8):

Convergence of the MBER beamforming to obtain the optimum weights on the Pre-FFT performance.

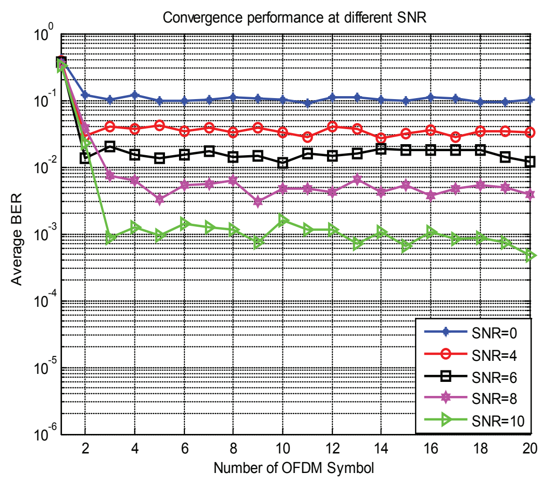

Fig. (9):

Convergence performance at different SNR.

CONCLUSION:

In this paper, we studied that MBER beamformer for Pre-FFT OFDM adaptive antenna array. A multipath (frequency selective fading) channel model is considered. The MBER algorithm and MMSE algorithm are compared. The MBER beamformer has advantageous characteristics such as better BER performance, less computational complexity and shorter training symbols.

REFERENCES:

1. C. K. Kim, K. Lee, and Y. S. Cho, "Adaptive beamforming algorithm for OFDM systems with antenna arrays", IEEE Trans. on Consumer Electronics, Vol. 64, No.4, Nov. 2000, pp. 1052-1058.

2. Lingyan Fan, Haibin Zhang and Chen He," Minimum bit error beamforming for Pre-FFT OFDM adaptive antenna array", IEEE International Conference, 28-25 Sept. 2005.

3. Saeid Seydnejad and Sadegh Akhzari "A combined time-frequency domain beamforming method for OFDM systems", 2010 International ITG Workshop on Smart Antennas, IEEE, 23-24 Feb. 2010.

4. S. Chen, N. N. Ahmad, and L. Hanzo, "Adaptive minimum bit-Error rate beamforming", IEEE Trans. on Wireless Comm., Vol. 4, No. 2 March 2005, pp.341-348.

5. T. A. Samir, S. Elnoubi, and A. Elnashar, "Class of minimum bit error rate algorithms," in Proc. 9th ICACT, Feb. 12–14, 2007 vol. 1, pp. 168–173.

6. Said Elnoubi, Waleed Abdallah, Mohamed M. M. Omar, "Minimum bit error rate beamforming combined with space-time block coding", The International Conf. on Com. and information Tech.-ICCIT 2011, March 2011-Aqaba, Jordon.

7. Z. Lei and P. S. Chin, "Post and Pre-FFT beamforming in an OFDM system" Proc. of IEEE Vehicular Tech. Conf., pp. Vol. 1, 17-19 May 2004, pp.39-43.

8. Shinsuke Hara, Montree Budsabathon, and Yoshitaka Hara, "A Pre-FFT OFDM adaptive antenna array with eigenvector combining", 2004 IEEE International Conference on comm. Vol. 4, June 2004, pp. 2412 - 2416.

9. J. Via´ , V. Elvira, I. Santamari´a, and R. Eickhoff, "Minimum BER beamforming in the RF domain for OFDM transmissions and linear receivers," in IEEE Int. Conf. on Acoustics, speech and signal Proc. (ICASSP 2009), Taipei, Taiwan, Apr. 2009.

10. Chong-Yung Chi, Chun-Hsien Peng, Kuan-Chang Huang, Teng-Han Tsai and Wing-Kin Ma ," A block-by block post-FFT multistage beamforming algorithm for multiuser OFDM systems based on subcarrier averaging", IEEE Trans. on wireless comm. Vol.7, No. 8, August 2008, pp.3238-3251.